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Composing Cardinal Direction Relations Based on Interval Algebra*

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Abstract Direction relations between extended spatial objects are important commonsense knowledge. Skiadopoulos proposed a formal model for representing direction relations between compound regions (the finite union of simple regions), known as SK-model. It perhaps is currently one of most cognitive plausible models for qualitative direction information, and has attracted interests from artificial intelligence and geographic information system. Originating from Allen first using composition table to process time interval constraints; composing has become the key technique in qualitative spatial reasoning to check the consistency. Due to the massive number of basic directions in SK-model, its composition becomes extraordinary complex. This paper proposed a novel algorithm for the composition. Basing the concepts of smallest rectangular directions and its original directions, it transforms the composition of basic cardinal direction relations into the composition of interval relations corresponding to Allen's interval algebra. Comparing with existing methods, this algorithm has quite good dimensional extendibility, that is, it can be easily transferred to the tridimensional space with a few modifications.

Key words: cardinal direction relation; interval algebra; composing; qualitative spatial reasoning

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1 Introduction

Spatial representation and reasoning play an essential role in human activities. Although the quantitative approaches can provide the most precise information, the numerical information is often unnecessary or unavailable at the human level. Qualitative approach for spatial reasoning, known as qualitative spatial reasoning (QSR), becomes a promising way to process spatial information at this level and has prevailed in artificial intelligence (AI), geographical information systems (GIS), database and multimedia communities for over three decades[1,2]. Consequently dozens of formalisms of spatial relations have been proposed topological, directional, distance and

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combined. *Topological* relations describe how the boundaries, the interiors and the exteriors of two objects are related. For instance, if a, b and c are spatial entities then a *includes* b and b *partially overlaps* c are the topological constraints[3−5]. Directional (or orientation) relations describe where the object is placed relative to another[6−19]. For instance a is placed on the *north* of b or b lies on the *north* and *northeast* of c are the directional constraints. The distance relations describe the relative distance between two spatial entities[17−19]. For instance a is *close to* b and b is *far from* c are the distance constraints. Finally the combined spatial relations usually involve different aspects of spatial relations, such as, directional and distance[17−19], topological and size[20], topological and directional[21−22], etc.

In this paper, we concentrate on *cardinal direction relations*. Several models capturing cardinal directions have been proposed in the literature[6−19]. Most models approximate the reference object by a point or its minimal bounding rectangle (MBR). Given two spatial objects, the representative point or the MBR divide the space around reference object into a number of mutually exclusive areas, the cardinal direction relation is the distribution of target object on these areas. Our start point is the model of Skiadopoulos (*SK-model*)[13−19] which is the currently one of most cognitive plausible models for qualitative reasoning with cardinal directions.

We focus on the problem of composing cardinal direction relations. Originating from Allen[23] first using composition table to process temporal constraints, it has become a key technique in QSR. Particularly, the consistency-based and existential definitions of the composing operator have attracted the interest of many researchers[24−28]. Typically, it is used as a mechanism for inferring new relations from existing ones and identifying classes of relations that have a tractable consistency problem. Due to the massive number of SK-model relations (511 basic relations), the composing operation becomes quite complex. In this paper we proposed a generative way, but not the consistency checking methods[14,16], *i.e.* to compute all possible results of the composition directly. Comparing with existing generative methods[13,15], it has nice dimensional expandability. *i.e.*, the given algorithm can be easily extended to tridimensional space with a little modification.

The following paper is structured as follows. Section 2 and section 3 introduce the basic definitions of SK-model and interval algebra. Section 4 builds the correlations between two formalisms. Section 5 explains the detail composing method and its proof. Section 6 is the extension and conclusions.

2 SK-model

Considering Euclidean space $\mathbb{R}^2$, regions are defined as non-empty and bounded sets of points in $\mathbb{R}^2$. Let a be a region, the *greatest lower bound* or the *infimum* of the *projection* of region $a$on the x-axis (respectively y-axis) is denoted by $inf_x(a)$ (respectively $inf_y(a)$). The *least upper bound* or the *supremum* of the *projection* of region $a$on the x-axis (respectively y-axis) is denoted by $sup_x(a)$ (respectively $sup_y(a)$). The *MBR* of a region a, denoted by $MBR(a)$, is the box formed by the straight lines $x= inf_x(a)$, $x= sup_x(a)$, $y= inf_y(a)$ and $y= sup_y(a)$ (see Fig.1).

Two different types of regions are considered throughout the paper.

Simple regions are the regions homeomorphic to the closed unit disk (*i.e.*, the set (x, y): $x^2 + y^2 \leqslant 1$). The set of these regions will be denoted by *REG*. Regions in

REG are closed, connected and has connected boundaries.

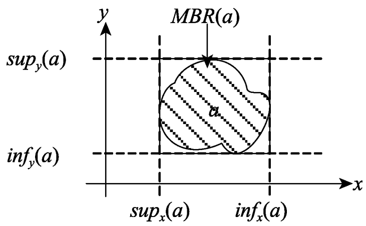

Figure 1. A region and its MBR

Compound regions can be seen as the finite union of simple regions. *i.e.*, A region $a$belongs to set REG^* *iff* there exists a finite set of regions $a_1,\ldots,a_n \in REG$ such that $a = a_1\cup\ldots\cup a_n$. Notice that regions in REG^* can be disconnected and may have holes.

As shown in Fig.2, a, $b_1, b_2$and$b_3$are simple regions included in REG; $b = b_1 \cup b_2 \cup b_3$ is a compound region and in REG^*. Obviously, $REG\subset REG^*$ holds.

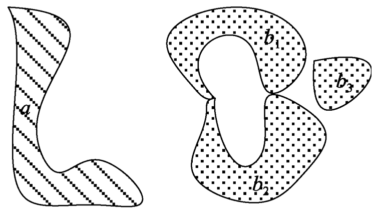

Figure 2. Simple region and compound region

A cardinal direction relation is a binary relation involving a target object and a reference object, and a symbol that is a non-empty subset of nine atomic relations whose semantics are motivated by the nine tiles divided by the MBR of the reference object b: northwest-$NW(b)$, north-$N(b)$, northeast-$NE(b)$, west-$W(b)$, east-$E(b)$, southwest-$SW(b)$, south-$S(b)$, southeast-$SE(b)$ and same-$O(b)$, *i.e.*, the MBR of b.. We have following definitions of atomic directions.

$dir(a,b) = O$ *iff* $inf_x(b)\leqslant inf_x(a)\wedge sup_x(a)\leqslant sup_x(b)\wedge inf_y(b)\leqslant inf_y(a)\wedge sup_y(a)\leqslant sup_y(b)$

$dir(a,b) = W$ *iff* $sup_x(a)\leqslant inf_x(b)\wedge inf_y(b) \leqslant inf_y(a)\wedge sup_y(a) \leqslant sup_y(b)$

$dir(a,b) = E$ *iff* $sup_x(b) \leqslant inf_x(a)\wedge inf_y(b)\leqslant inf_y(a)\wedge sup_y(a) \leqslant sup_y(b)$

$dir(a,b)=NW$ *iff* $sup_x(a) \leqslant inf_x(b)\wedge sup_y(b) \leqslant inf_y(a)$

$dir(a,b) = N$ *iff* $sup_x(a) \leqslant sup_x(b)\wedge sup_y(b) \leqslant inf_y(a)\wedge inf_x(b)\leqslant inf_x(a)$

$dir(a,b)=NE$ *iff* $sup_x(b) \leqslant inf_x(a)\wedge sup_y(b) \leqslant inf_y(a)$

$dir(a,b)=SW$ *iff* $sup_x(a)\leqslant inf_x(b)\wedge sup_y(a) \leqslant inf_y(b)$

$dir(a,b) = O$ *iff* $inf_x(b)\leqslant inf_x(a)\wedge sup_x(a)\leqslant sup_x(b)\wedge inf_y(b)\leqslant inf_y(a)\wedge sup_y(a)\leqslant sup_y(b)$

$dir(a,b) = S$ *iff* $inf_x(b) \leqslant inf_x(a)\wedge sup_x(a) \leqslant sup_x(b)\wedge sup_y(a) \leqslant inf_y(b)$

$dir(a,b)=SE$ *iff* $sup_y(a) \leqslant inf_y(b)\wedge sup_x(b)\leqslant inf_x(a)$

There are 2^9-1=511 basic directions can be realized over REG^*, denoted by D^*; and only 218 basic relations[12], denoted by D, can be satisfied in REG. $D(D^*)$ constitutes a JEPD (Jointly Exhaustive and Pairwise Disjoint) list of all possible relations which can hold over $REG(REG^*)$. The set of cardinal direction relation is defined as the power set of $D(D^*)$. Each element of $2^D(2^{D^*})$ can be seen as the union of its basic relations, which is used to describe the indeterminate information, *e.g.* $dir(a, b)$={$NW{:}N$, $NW{:}W$} means the direction a with respect to b is $NW{:}N$ or $NW{:}W$.

As shown in Fig.3, the direction a with respect to b is denoted by $dir(a, b)= W{:}SW{:}S$ and $dir(c, b) = E : S$ which can only be satisfied over REG^*. Obviously, $D \subset D^*$holds.

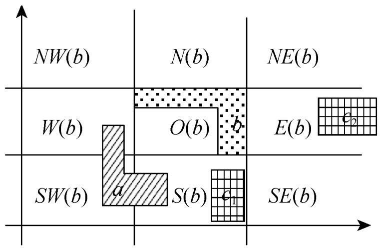

Figure 3. Pictorial examples of basic cardinal direction

3 Interval Algebra

Interval algebra (IA) was proposed by Allen[23] for temporal reasoning. According to the different order relations between the temporal intervals' endpoints, it defines 13 basic interval relations, which constitutes a JEPD list of all possible relations which can hold between temporal intervals, denoted by A_{int}={p, pi, m, mi, e, d,di, s, si, f, fi, o, oi} as shown in Table 1, where *sup* and *inf* represent the start and end point of the interval separately. Each element of the power set 2^{Aint} can be seen as the union of basic relations, which is used to describe the indeterminate information. *e.g.* {e, m} means the possible relations between the intervals are equals or meets.

Table 1 Basic interval relations(a is ⊢——⊣, b is ⊢– –⊣)

relation(arb)	symbol	definition	pictorial example	inverse
precedes	p	$sup(a) < inf(b)$		pi
meets	m	$sup(a) = inf(b)$		mi
overlaps	o	$inf(a) < inf(b) < sup(a) < sup(b)$		oi
starts	s	$inf(b) = inf(a) \wedge sup(a) < sup(b)$		si
during	d	$inf(b) < inf(a) \wedge sup(a) < sup(b)$		di
finishes	f	$inf(b) < inf(a) < sup(a) = sup(b)$		fi
equal	e	$inf(b) = inf(a) \wedge sup(a) = sup(b)$		e

When dealing with planar or cubic space, IA can be extended to rectangle algebra[29] and block algebra[30] which describe the spatial configurations by listing the relation for each coordinate separately, while it can only describe rectangles or blocks whose sides are parallel to the orthogonal basis. Here we only consider the

Euclidean space R^2, the relation between the given rectangles must be in the rectangle relations denoted by $A_{rec}=\{(r_x,\ r_y)|\quad r_x,\ r_y \in A_{int}\}$[24]. As shown in Fig.4, the relation between rectangle a and b is the pair (o, p).

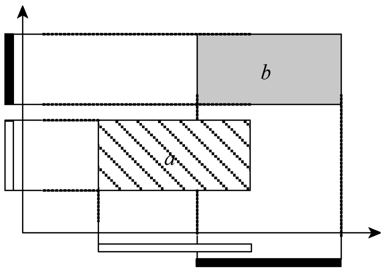

Figure 4. Picture of basic rectangle relation (o, p)

4 Correlations between Cardinal Direction Relations and Interval Algebra

According to the definition of cardinal direction relation, we know that the shape of reference object b do not influence the direction, *i.e.* the $MBR(b)$ and b are equal when as a reference object. So if we only consider the MBR of target object a, then different direction relations $dir(a,\ b)$ have the same direction relation of $dir(MBR(a),$ $MBR(b))$. As shown in Fig.5, although the direction between a and b is different, it has the same rectangular direction. We have following definitions.

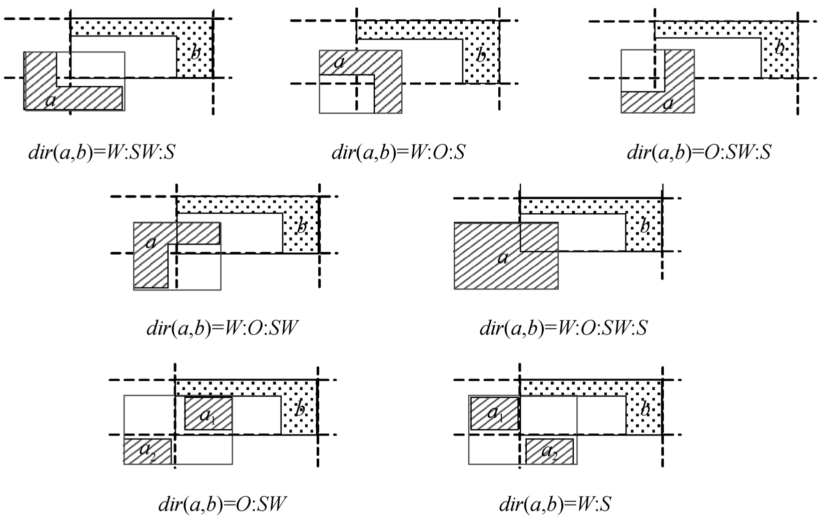

Figure 5. Illustration of SRD and ORG

Definition 1. Basic direction r is rectangular *iff* there exists two rational rectangles a and b such that $dir(a,b) = r$ is satisfied; otherwise, it is non-rectangular.

Example 1. All atomic directions are rectangular. $W{:}O$ is rectangular while $W{:}SW{:}S$ is not, for no rectangles can satisfy this relation.

Definition 2. Basic direction $r_1 = r_{11}{:}\ldots{:}r_{1m}$ includes basic relation $r_2 = r_{21}{:}\ldots{:}r_{2n}$ *iff* $\{r_2,\ldots,r_{2n}\}\subseteq\{r_{11},\ldots,r_{1m}\}$, denoted by $r_2 \subseteq r_1$.

Example 2. $W{:}O$(properly) includes atomic relations W and O.

Definition 3. Basic direction r_1 is the smallest rectangular direction (SRD) of r_2, denoted by $SRD(r_2)$, *iff* it is rectangular and the smallest direction includes r_2.

Definition 4. Basic directions whose smallest rectangular direction is r are called the original directions of r, denoted by $ORG(r)$. $ORG(r)$ might be a single basic direction or a set of basic directions.

Example 3. $ORG(W)$={W}, $ORG(W{:}O{:}SW{:}S)$={$W{:}SW{:}S$, $SW{:}O{:}S$, $O{:}SW{:}S$, $W{:}O{:}SW$, $W{:}O{:}SW{:}S$, $SW{:}S$, $O{:}SW$ } over *REG**, when restricted to simple regions, $SW{:}S$ and $O{:}SW$ can not be satisfied as Fig.5 shows.

According to the definitions of cardinal direction relation and the interval relations; for arbitrary rectangles satisfying any rectangular direction, the projections of these rectangles on each axis must satisfy a certain interval relation as Table 2 shows.

Table 2 Six groups of interval relations

Interval relation	Projection relation $_{\pi=\{x,y\}}$
$\{p, m\}$	$sup_\pi(a) \leqslant inf_\pi(b)$
$\{o, fi\}$	$inf_\pi(a) < inf_\pi(b) < sup_\pi(a) \leqslant sup_\pi(b)$
$\{e, d, s, f\}$	$inf_\pi(b) \leqslant inf_\pi(a) \wedge sup_\pi(a) \leqslant sup_\pi(b)$
$\{pi, mi\}$	$sup_\pi(b) \leqslant inf_\pi(a)$
$\{oi, si\}$	$inf_\pi(b) < inf_\pi(a) < sup_\pi(b) \leqslant sup_\pi(a)$
$\{di\}$	$inf_\pi(a) < inf_\pi(b) \wedge sup_\pi(b) < sup_\pi(a)$

As shown in Fig.6, a and b satisfy the same rectangular direction relation, the rectangle relations formed by their projections are different.

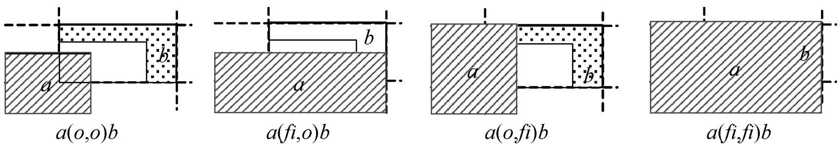

Figure 6. Rectangle relations corresponding to $W{:}O{:}SW{:}S$

Let r be a rectangular direction, $REC(r)=REC_x(r)\times REC_y(r)$ is the corresponding rectangle relation of r according to Table 2. For example, if $r=W{:}O{:}SW{:}S$, then $REC(r)=\{o, fi\}\times\{o, fi\}$. Contrariwise, for a rectangle relation T, its corresponding rectangular direction is $CD(T) = \cup\{CD(t)|t \in T\}$. *e.g.*, $T=\{s, o\}\times\{f, o\}$, $CD(T)=CD((s, f)) \cup CD(o, f)=\{O, W{:}O\}$.

Based on above definitions, the set of basic directions can be divided into 6^2=36 equivalence classes, as shown in Table 3. Each element in each class is equal in its *SRD*, while each basic rectangle relation corresponds to a rectangular direction which is the *SRD* of a set of basic directions. Thus we can establish the mapping between basic cardinal direction and rectangle relations.

Table 3 Rectangle relations of rectangular direction

$REC_x(r)$ / $REC_y(r)$	$\{p,m\}$	$\{o,fi\}$	$\{e,d,s,f\}$	$\{si,oi\}$	$\{pi,mi\}$	$\{di\}$
$\{p,m\}$	*SW*	*SW:W*	*W*	*NW:W*	*NW*	*SW:W:NW*
$\{o,fi\}$	*SW:S*	*SW:S:W:O*	*W:O*	*NW:N: W:O*	*NW:N*	*NW:N:W:O: SW:S*
$\{e,d,s,f\}$	*S*	*S:O*	*O*	*N:O*	*N*	*N:O:S*
$\{si,oi\}$	*S:SE*	*S:SE:O:E*	*O:E*	*N:NE: O:E*	*N:NE*	*N:NE:O:E:S:SE*
$\{pi,mi\}$	*SE*	*E:SE*	*S:SE*	*NE:E*	*NE*	*NE:E:SE*
$\{di\}$	*SW:S:SE*	*W:O:E:SW:S:SE*	*W:O:E*	*NW:N:NE:W:O:E*	*NW:N:NE*	*NW:N:NE:W: O:E:SW:S:SE*

5 Composing

Originating from Allen[23] first using composition table to process time interval constraints; the notion of composition plays a very important role in qualitative spatial reasoning. It has become the key technique in QSR to check the consistency.

Definition 5. Let r_1 and r_2 be two basic directions, their composition, denoted by $r_1 \circ r_2$is a set of basic directions satisfying: for each $t \in r_1 \circ r_2$, there exist region a, b, $c \in REG^*$ such that $dir(a,\ b) = r_1 \wedge dir(b,\ c) = r_2 \wedge dir(a,c) = t$ is satisfied.

Theorem 1. For arbitrary basic directions r_1, $r_2 \in D^*$, $r_1 \circ r_2 = r_1 \circ SRD(r_2)$ holds.

Proof: For $\forall t \in r_1 \circ r_2$, there exist a, b, $c \in REG^*$, such that

$$dir(a,\ b) = r_1 \wedge dir(b,\ c) = r_2 \wedge dir(a,\ c) = t.$$

Then according to the definitions of cardinal directions and SRD,

$$dir(a,\ MBR(b)){=}r_1 \wedge dir(MBR(b),\ MBR(c)){=}SRD(r_2) \wedge dir(a,\ MBR(c)){=}t$$

So, $t \in r_1 \circ SRD(r_2)$ holds.

Conversely, $\forall t \in r_1 \circ SRD(r_2)$, there exists region a, b, $c \in REG^*$, such that

$$dir(a,\ b) = r_1 \wedge dir(b,\ c){=}SRD(r_2) \wedge dir(a,\ c) = t$$

Assume $r_2 = r_{21}{:}\ldots{:}r_{2k}$, form region $d{=}MBR(b) \cap r_{21}(c) \cap \ldots \cap r_{2k}(c)$ satisfying $MBR(b){=}MBR(d)$. Therefore,

$$dir(a,\ d) = r_1 \wedge dir(d,\ c) = r_2 \wedge dir(a,\ c) = t$$

so, $t \in r_1 \circ r_2$ is satisfied. □

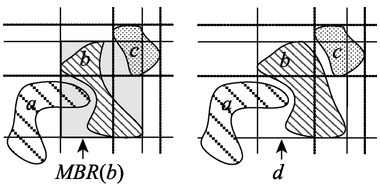

Figure 7. Proof of Theroem 1

Lemma 1. Let $r_1$be atomic direction, r_2 be a rectangular direction and $t \in r_1 \circ r_2$, then the following implication holds

$$(\forall b,\, c \in REG^*)\ (dir(b,c) = r_2 \rightarrow \exists a(dir(a,\, b) = r_1 \wedge dir(a,c) = t).$$

Proof: If $\forall b,\, c \in REG^*$ satisfying $dir(b,\, c) = r_2$, then assume $\beta_1(c),\ldots,\beta_9(c)$ are the 9 tiles divided by $MBR(c)$, it must exist a maximal subset $\{\delta_l(c),\ldots,\, \delta_m(c)\} \subseteq \{\beta_1(c),\, \ldots,\, \beta_9(c)\}$, such that $i \in \{l\ldots m\}$,

$$\delta_i(c) \cap r_1(b) \neq \emptyset \wedge \forall \delta_j(c) \in \{\beta_1(c),\, \ldots,\, \beta_9(c)\} \backslash \{\delta_l(c),\ldots,\, \delta_m(c)\}\ \delta_j(c) \cap r_1(b) = \emptyset.$$

Let $t = t_1{:}\ldots{:}t_k$ and $\delta{=}\delta_l{:}\ldots{:}\ \delta_m$, if $t \in r_1 \circ r_2$ then $t \subseteq \delta$ holds, *i.e.*,

$$\exists a \in REG^* \text{ such that } a \in r_1(b) \wedge a \cap t_i(c) \neq \emptyset,\ 1 \leqslant i \leqslant k.$$

Otherwise, there exists t_p, $1 \leqslant p \leqslant k$ such that

$$t_p(c) \in \{\beta_1(c),\, \ldots,\, \beta_9(c)\} \backslash \{\delta_l(c),\ldots,\, \delta_m(c)\} a \cap t_p(c) \neq \emptyset$$

holds. Since $a \in r_1(b)$ then $r_1(b) \cap t_p(c) \neq \emptyset$which contradicts $r_1(b) \cap t_p(c) = \emptyset$.

To facilitate the illustration, $\sigma(s_1,..,s_m)$ is the shortcut for the set of all valid basic directions that can be constructed by cross joining the set $s_1,\ldots,s_m$, such as $\sigma(\{O\},\{W{:}NW,\ W\}) = \{O{:}W{:}NW,\ O:W\}$.

Theorem 2. Let $r_1 = r_{11}{:}\ldots{:}r_{1k}$ be basic direction and r_2 be rectangular directionthen $r_1 \circ r_2 = \sigma(r_{11} \circ r_2,\ldots,r_{1k} \circ r_2)$ holds.

Proof: Let $t \in r_1 \circ r_2$, there exist $a,\, b,\, c \in REG^*$ such that

$$dir(a,\, b) = r_1 \wedge dir(b,\, c) = r_2 \wedge dir(a,\, c) = t.$$

Since $dir(a,\, b) = r_1$, there exist$a_1,\ldots,a_k \in REG^*$ such that

$$a = a_1 \cup \ldots \cup a_k \text{and } dir(a_1,b) = r_{11} \wedge \ldots \wedge dir(a_k,b) = r_k \wedge dir(b,c) = r_2 \wedge dir(a,c) = t$$

Therefore, there exist basic relations $t_1,\ldots,t_k$ and $t \in \sigma(t_1,\, \ldots,t_k)$ such that $dir(a_i,\, c) = t_i$, $1 \leqslant i \leqslant k$ are satisfied. So

$$dir(a_i,\, b) = r_{1i} \wedge dir(b,\, c) = r_2 \wedge dir(a_1,\, c) = t_i \quad i.e.,\ t_i \in r_{1i} \circ r_2.$$

It means $t \in \sigma(r_{11} \circ r_2,\ldots,r_{1k} \circ r_2)$.

Conversely, $\forall t \in \sigma(r_{11} \circ r_2,\ldots,r_{1k} \circ r_2)$, assume $t{=}\sigma(t_1,\ldots,t_k)$ and $t_i \in r_{1i} \circ r_2$, $1 \leqslant i \leqslant k$. There exist $a_i,\, b,\, c \in REG^*$ such that

$$dir(a_i,b) = t_{1i} \wedge dir(b,c) = r_2 \wedge dir(a_i,c) = t_i$$

by Lemma 1. Therefore, let $a = \cup_i a_i$, then

$$dir(a,\, b) = r_1 \wedge dir(b,\, c) = r_2 \wedge dir(a,\, c) = t$$

is satisfied, i.e., $t \in r_1 \circ r_2$. □

Theorem 3. Let r_1 be an atomic direction and r_2 be a rectangular direction, then

$$r_1 \circ r_2 = \{\cup_t ORG(t):\ t \in CD(REC(r_1) \circ REC(r_2))\}$$

Proof: $\forall u \in r_1 \circ r_2$, there must exist a, b, $c \in REG^*$, such that

$$dir(a,\ b) = r_1 \wedge dir(b,\ c) = r_2 \wedge dir(a,\ c) = u$$

Since $r_1$and r_2 are rectangular,

$$dir(MBR(a),\ MBR(b)){=}r_1 \wedge dir(MBR(b),\ MBR(c)){=}r_2$$

From the point of rectangle algebra

$$MBR(a)\ REC(r_1)\ MBR(b) \quad \wedge\ MBR(b)\ REC(r_2)\ MBR(c)$$

So $dir(MBR(a),\ MBR(c)) \in REC(r_1) \circ REC(r_2)$, *i.e.*, $SRD(u) \in CD(REC(r_1) \circ REC(r_2))$. Let $t{=}SRD(u)$ then $u \in ORG(t)$.

Conversely, $\forall t \in CD(REC(r_1) \circ REC(r_2))$, there must exist rectangle a, b and $c \in REG^*$ such that

$$dir(a,\ b) = r_1 \wedge dir(b,\ c) = r_2 \wedge dir(a,\ c) = t.$$

Then assume $\forall u = u_1{:}\ldots{:}u_k \in ORG(t)$, form a rectangle $a_0 = a \cap u_1(c) \cap \ldots \cap u_k(c)$, we have

$$dir(a_0,\ b) = r_1 \wedge dir(b,\ c) = r_2 \wedge dir(a_0,\ c) = u\ .$$

Therefore $u \in r_1 \circ r_2$ holds. □

To sum up, for arbitrary basic relation $r_1 = r_{11}{:}\ldots{:}r_{1k}$ and r_2, following equations must hold:

-$r_1 \circ r_2 = r_1 \circ SRD(r_2)$

-$r_1 \circ r_2 {=} \sigma(r_{11} \circ SRD(r_2), \ldots, r_{1k} \circ SRD(r_2))$

-$r_1 \circ r_2 {=} \sigma(ORG(t_1), \ldots,\ ORG\ (t_k))$where $t_i{=}CD(REC(r_{1i}) \circ REC(SRD(r_2)))$, $1 \leqslant i \leqslant k$.

Then we get following composing algorithm *CDCom.*

Algorithm. CDComp

Input: basic cardinal direction r, $t \in D*$

Output: the result of $r \circ t$, the set R

1. Let $q{=}SRD(t)$, compute the corresponding rectangle relation $REC(q)$
2. Assume $r = r_1{:}\ldots{:}r_k$, compute the rectangle relation of r_i, $REC(r_i)$, where $i{=}1, \ldots, k$
   For $i{=}1,\ldots,k$
   Compute $P_i{=}\ REC(r_i) \circ REC(q)$ and $CD(P_i)$
3. For $i{=}1,\ldots,k$
   $T_i = \emptyset$
   For each d$\in CD(P_i)$
   Compute $T_i = T_i \cup ORG(d)$
4. *Return* $R{=}\sigma(T_1, \ldots,\ T_k)$. □

Example 4. Compute the composition of $r = W{:}O$, $t = W{:}SW{:}S$

1. Let $q=SRD(t) = W{:}O{:}SW{:}S$, then $REC(q)=\{o, fi\}\times\{o, fi\}$ according to Table 3.

2. The atomic directions r including are W and O, then we have:

$REC(W)=\{p, m\}\times\{e, d, s, f\}$,

$P_1=REC(W)\circ REC(q)= \{p\}\times\{p, m, o, fi, e, d, s, f\}$

$CD(P_1)= \{SW, W{:}SW, W\}$

$REC(O)=\{e, d, s, f\}\times\{e, d, s, f\}$

$P_2= REC(O)\circ REC(q)=\{p, m, o, fi, e, d, s, f\}\times\{p, m, o, fi, e, d, s, f\}$,

$CD(P_2)=\{SW, W{:}SW, W, SW{:}S, SW{:}S{:}O{:}W, W : O, S, S : O, O\}$

3. As except $SW{:}S{:}O{:}W$, the original directions of other basic directions are themselves. From Theorem 3, we have:

$T_1=\{SW, W{:}SW, W\}$,

$T_2=\{SW, W{:}SW, W, SW{:}S, SW{:}S{:}O{:}W, S : O : W, SW{:}O{:}W, SW{:}S{:}W, SW{:}S{:}O, W : O, S, S : O, O, W{:}S, O{:}SW\}$

4. The final result $R=\sigma(T_1,T_2)=\{W{:}O{:}SW{:}S, W{:}O{:}SW, W{:}O{:}S, W{:}SW{:}S, O{:}SW{:}S, W{:}O, SW{:}S, W{:}SW, W, SW, SW{:}O, W{:}S\}$ as Fig.8 shows orderly.

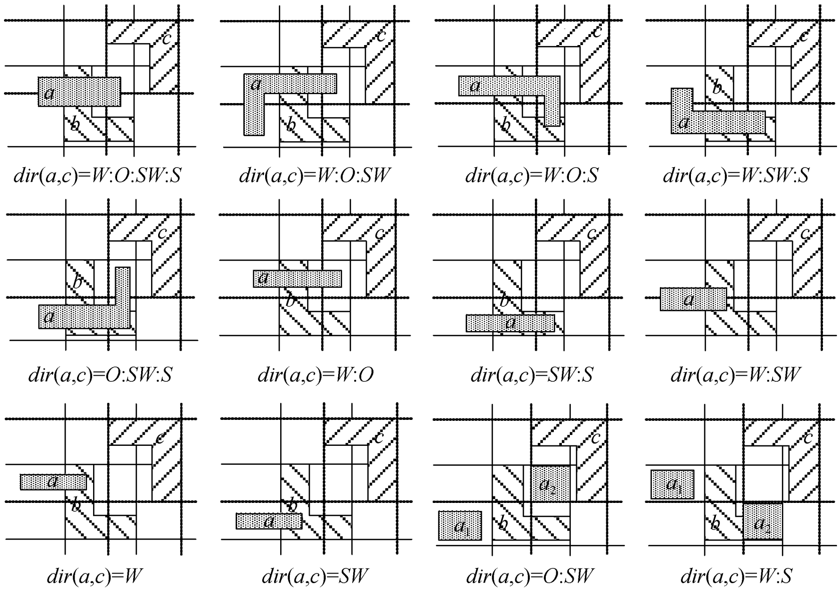

Figure 8. Pictorial examples of $W{:}O\circ W{:}SW{:}S$

6 Conclusions

Basing on the concepts of smallest rectangular directions and the original directions in SK-model, this paper translates the composition of basic cardinal directions into the composition of interval relations and gives the proof. Comparing with existing method, the given algorithm has quite good dimensional extendibility. The similar results can be easily attained in tridimensional space just like the extension from interval algebra to block algebra[30], as Fig.9 shows the tridimensional model.

While two issues should be mentioned here are that algorithm *CDComp* can only process the consistency-based composition and work correctly over compound regions *REG**.

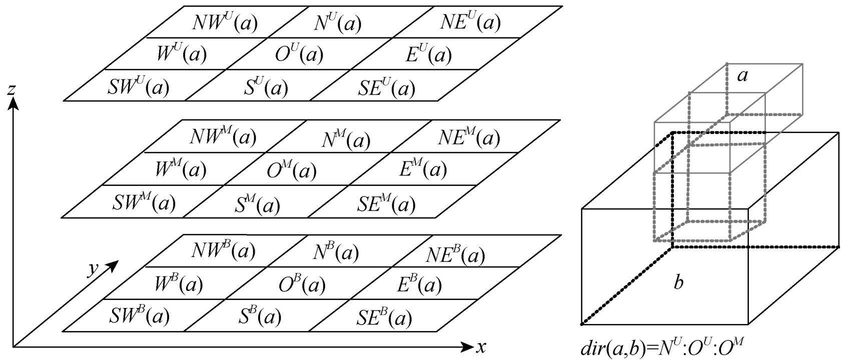

Figure 9. Tridimensional cardinal direction relations

Algorithm *CDComp* gives consistency-based composition (weak composition), which can be easily derived from the proofs of Theroem1∼4. For arbitrary direction r and s:$r \circ_w s$={ t_1,...,t_m}*where$\forall a \forall b \forall c((dir(a,b) = r \wedge dir(b,c) = s) \rightarrow (dir(a,c) = t_1 \vee ... \vee dir(a,c) = t_m))$ holds. $N \in N \circ_w N$, as the left figure of Fig.10 shows there exist regions satisfy the above formula. While for the existence-based composition (strong composition), $\forall dir(a,c) \in r \circ_s s \leftrightarrow \exists b$: $dir(a,b) = r \wedge dir(b,c) = s$. $N \notin N \circ_s N$, for a meets c from norh, no region b satisfy $dir(a,b) = N \wedge dir(b,c) = N \wedge dir(a,c) = N$ as the right figure shows.

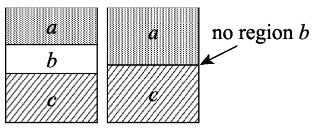

Figure 10. Explanation of weak composition

Algorithm *CDComp* can only give correct results over *REG**. The main reason lies in the proof of Theorem 2. When forming new region $a = \cup_i a_i$, there is no condition to limit a as a simple region, so it can only ensure a be a compound region. Example 6 gives the instance. According to Theorem 3

$NW \in NW \circ NW{:}N{:}W \quad NW \in N \circ NW{:}N{:}W, \quad NE \in NE \circ NW{:}N{:}W$

$W \in W \circ NW{:}N{:}W, \quad O : E \in E \circ NW{:}N{:}W$

as shown in Fig.11, therefore we have $NW{:}NE{:}W{:}O{:}E \in NW{:}N{:}NE{:}W{:}E \circ NW{:}N{:}W$ based on Theorem 2. But there is no simple regions such that $dir(a,b)=NW{:}N{:}NE{:}W{:}E \wedge dir(b,c)=NW{:}N{:}W \wedge dir(a,c)=NW{:}NE{:}W{:}O{:}E$ holds.

Example 6. let $r= NW{:}N{:}NE{:}W{:}E$, $t=NW{:}N{:}W$ then $s=NW{:}NE{:}W{:}O{:}E \in r \circ t$ can only be satisfied in *REG**

In this paragraph, we use$\circ_w$and $\circ_s$ to differentiate weak and strong composition, while out of this paragraph $\circ$ still denotes the weak composition of directions.

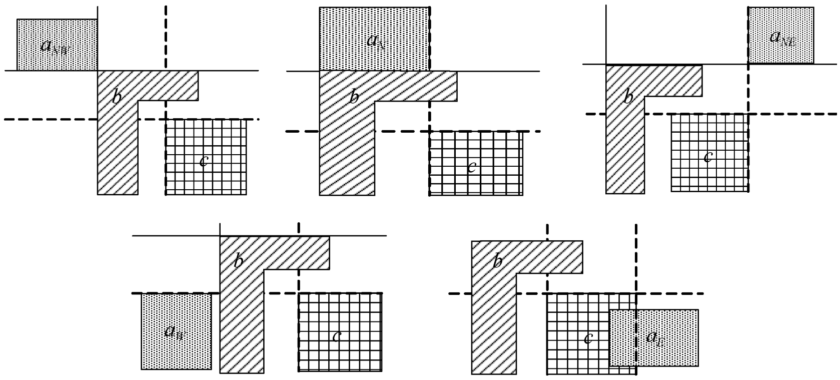

Figure 11. Illustration of evidences of Example 6

Most potential applications of QSR involve different aspects of space. So reasoning with multi-aspect spatial information has become the focus of QSR. Some work has been done in this issue[17−22] but it mainly concentrates on the two aspects of space such as topology and direction, topology and size, distance and direction. It lacks the integrative reasoning over three or more aspects of spatial relations. Due to the good extendibility of interval algebra which can represent both directional and topological information in one calculus; it is worth introducing other type information into it to realize the integrative reasoning over three or more aspects of spatial relations.

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