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Channel Coupling: Entanglement Preservation via Unentangled Correlations

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Abstract Give two quantum channels acting on different systems, we provide a natural and powerful method for coupling them by virtue of correlations between local environments. As an application, we demonstrate, through very simple examples, that separable (unentangled) correlations are a useful resource for preserving entanglement, which otherwise would be completely destroyed by entanglement breaking channels if environmental correlations were absent. This reveals a mechanism for engineering environments in fighting decoherence.
Key words: channel coupling; entanglement breaking; correlations; entanglement protection

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1 Introduction

Quantum systems are usually coupled with environments and thus subjected to decoherence, which is ubiquitous in the interaction between classical and quantum[1,2]. Combating decoherence is a vital and inescapable step in quantum information processing. There are various approaches to this issue such as error corrections and decoherence free subspaces[3–6], dynamical decoupling[7–9], environmental engineering[10,11], etc.

A notorious manifestation of decoherence is the entanglement degradation, and more dramatically, entanglement sudden death[12]. Many basic quantum tasks require entanglement and its preservation. In this respect, an intriguing and puzzling phenomenon is that separable correlations can be used for distributing entanglement[13–16], and in particular, for manipulating and restoring broken entanglement within the continuous variable Gaussian quantum information setup[17–19]. Motivated by these studies and other foundational considerations, here we explore a method for mitigating decoherence and preserving entanglement by exploiting separable environmental correlations. Through a very simple scheme, we

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highlight that for qubit systems, separable correlations are a resource for preserving entanglement and fighting decoherence. For this purpose, we first develop a convenient and powerful method for coupling channels (linear, completely positive and trace preserving maps on quantum states) on different systems which is of independent interest, investigate its fundamental properties, and then employ a curious effect of this coupling to manipulate entanglement.

2 Channel Coupling

Although there seem very few studies on how to couple different channels in a general way[19], in some sense, channel coupling is a generalization of quantum state coupling, which is essentially the subject of correlations. The latter is well studied [20–24], and can be employed in establishing channel coupling.

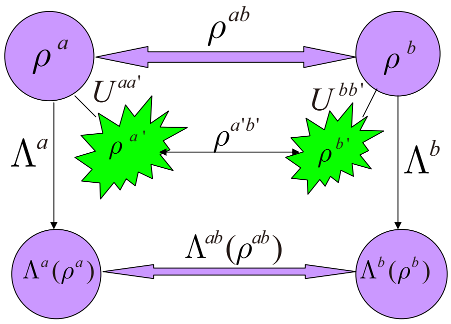

Figure 1. Entanglement preservation via separable (untangled) correlations. A state ρ^{ab}, shared between systems a and b, is subjected to channels Λ_a and Λ_b defined by Eqs. (1) and (2), respectively. Here $\rho^{a'}$ and $\rho^{b'}$ are two environmental states for a and b, respectively. For the joint action of the channels, there are two distinctive cases: (i) If the two channels are independent in the sense that the environments are independent, then the joint action $\Lambda^a \otimes \Lambda^b$ is defined by Eq. (3). (ii) If the environments are correlated in the joint environmental state $\rho^{a'b'}$ with reduced states $\rho^{a'} = \mathrm{tr}_{b'}\rho^{a'b'}$ and $\rho^{b'} = \mathrm{tr}_{a'}\rho^{a'b'}$, then the joint action, as a coupling of Λ^a and Λ^b, is Λ^{ab} defined by Eq. (4). The initial entanglement stored in ρ^{ab}, while broken by $\Lambda^a \otimes \Lambda^b$, may be partially preserved even if the environments are separably correlated (unentangled) and the coupled channel Λ^{ab} is effected instead of $\Lambda^a \otimes \Lambda^b$.

The precise setup is as follows. Consider two channels Λ^a and Λ^b on systems a and b, respectively. Apart from the Kraus sum representations, they can always be cast in the unitary representation forms as [25]

$$\Lambda^a(\rho^a) = \mathrm{tr}_{a'} U^{aa'}(\rho^a \otimes \rho^{a'})(U^{aa'})^\dagger, \tag{1}$$

$$\Lambda^b(\rho^b) = \mathrm{tr}_{b'} U^{bb'}(\rho^b \otimes \rho^{b'})(U^{bb'})^\dagger. \tag{2}$$

Here $U^{aa'}$ and $U^{bb'}$ are unitary operators on system-environments aa' and bb', respectively, with a' and b' local environments pertaining to systems a and b, respectively. Symbolically, $\Lambda^a = \{U^{aa'}, \rho^{a'}\}$, $\Lambda^b = \{U^{bb'}, \rho^{b'}\}$. It should be emphasized that the channels depend on both the unitary operators and the environmental states. The change of the environmental states alone may completely alter the channels, such as from entanglement breaking to partial entanglement preserving.

Now consider a bipartite state ρ^{ab} shared by two systems a and b with reduced states $\rho^a = \mathrm{tr}_b\rho^{ab}$, $\rho^b = \mathrm{tr}_a\rho^{ab}$. If the two channels act independently (the environments a' and b' are independent), then the joint action is defined by the (direct product) channel

$$\Lambda^a \otimes \Lambda^b := (I^a \otimes \Lambda^b)(\Lambda^a \otimes I^b) = (\Lambda^a \otimes I^b)(I^a \otimes \Lambda^b). \tag{3}$$

Here I^a and I^b are the identity channels on systems a and b, respectively. If the channel Λ^a or Λ^b is entanglement breaking[26,27], then clearly $\Lambda^a \otimes \Lambda^b$ is also entanglement breaking. In this case, the entanglement in the system state ρ^{ab} is destroyed by the channel action. The direct product channel $\Lambda^a \otimes \Lambda^b$ corresponds to the situation schematically depicted in Fig. 1 when the environmental states are uncorrelated, that is, the joint state of the two environments is $\rho^{a'b'} = \rho^{a'} \otimes \rho^{b'}$.

Now, the natural question arises as what will happen if the two environmental states $\rho^{a'}$ and $\rho^{b'}$ are correlated. Here we demonstrate that by coupling the environments a' and b', we may partially preserve the entanglement. This happens even if there is no entanglement between the environments: Separable correlations suffice. When the environments a' and b' are correlated in the composite state $\rho^{a'b'}$, we define the joint action

$$\Lambda^{ab}(\rho^{ab}) := \mathrm{tr}_{a'b'} W(\rho^{ab} \otimes \rho^{a'b'})W^\dagger \tag{4}$$

as a coupling of Λ^a and Λ^b. Here $W := U^{aa'} \otimes U^{bb'}$. It should be emphasized that the unitary operators $U^{aa'}$ and $U^{bb'}$ act on system-environments aa' and bb', respectively.

The channel coupling defined by Eq. (4) has the following remarkable properties:

(i) The reduced (marginal) channels of Λ^{ab} are Λ^a and Λ^b in the sense that

$$\mathrm{tr}_b\Lambda^{ab}(\rho^{ab}) = \Lambda^a(\rho^a), \quad \mathrm{tr}_a\Lambda^{ab}(\rho^{ab}) = \Lambda^b(\rho^b).$$

(ii) If $\rho^{a'b'} = \rho^{a'} \otimes \rho^{b'}$ is a product (uncorrelated) state, then $\Lambda^{ab} = \Lambda^a \otimes \Lambda^b$.

(iii) If both ρ^{ab} and $\rho^{a'b'}$ are separable (i.e., unentangled), then $\Lambda^{ab}(\rho^{ab})$ is separable.

(iv) If $\rho^{a'b'}$ is separable, then the entanglement in ρ^{ab}, as quantified by the concurrence $C(\cdot)$ (or any other entanglement monotone)[23,28], will decrease after the action of the channel Λ^{ab}, i.e., $C(\Lambda^{ab}(\rho^{ab})) \leqslant C(\rho^{ab})$.

Items (i) follows from direct manipulation of partial trace.

To prove (ii), noting that

$$(I^a \otimes \Lambda^b)(\rho^{ab}) = \mathrm{tr}_{b'}(\mathbf{1}^a \otimes U^{bb'})(\rho^{ab} \otimes \rho^{b'})(\mathbf{1}^a \otimes U^{bb'})^\dagger,$$

we have

$$\begin{aligned}
&\Lambda^a \otimes \Lambda^b(\rho^{ab}) \\
&= (\Lambda^a \otimes I^b)(I^a \otimes \Lambda^b)(\rho^{ab}) \\
&= \mathrm{tr}_{a'}(U^{aa'} \otimes \mathbf{1}^b)((I^a \otimes \Lambda^b)(\rho^{ab}) \otimes \rho^{a'})(U^{aa'} \otimes \mathbf{1}^b)^\dagger \\
&= \mathrm{tr}_{a'b'}(U^{aa'} \otimes U^{bb'})(\rho^{ab} \otimes (\rho^{a'} \otimes \rho^{b'}))(U^{aa'} \otimes U^{bb'})^\dagger
\end{aligned}$$

$$= \Lambda^{ab}(\rho^{ab}).$$

Here $\mathbf{1}^a$ and $\mathbf{1}^b$ are the identity operators on the Hilbert spaces of systems a and b, respectively.

To establish (iii), noting that both ρ^{ab} and $\rho^{a'b'}$ are separable, we can always write $\rho^{ab} = \sum_j u_j \rho_j^a \otimes \rho_j^b$ and $\rho^{a'b'} = \sum_k v_k \rho_k^{a'} \otimes \rho_k^{b'}$ with $\rho_j^a, \rho_j^b, \rho_k^{a'}$ and $\rho_k^{a'}$ states on systems a, b, a' and b', respectively, and $u_j, v_k \geqslant 0$. Put

$$\varrho_{jk}^a := \mathrm{tr}_{a'} U^{aa'}(\rho_j^a \otimes \rho_k^{a'})(U^{aa'})^\dagger,$$
$$\varrho_{jk}^b := \mathrm{tr}_{b'} U^{bb'}(\rho_j^b \otimes \rho_k^{b'})(U^{bb'})^\dagger,$$

then the final state is

$$\Lambda^{ab}(\rho^{ab}) = \sum_{jk} u_j v_k \varrho_{jk}^a \otimes \varrho_{jk}^b,$$

which is apparently separable.

To establish (iv), noting that when $\rho^{a'b'} = \sum_k v_k \rho_k^{a'} \otimes \rho_k^{b'}$ is separable, we have

$$\Lambda^{ab}(\rho^{ab}) = \sum_k v_k \Lambda_k^a \otimes \Lambda_k^b(\rho^{ab})$$

with

$$\Lambda_k^a(\rho^a) := \mathrm{tr}_{a'} U^{aa'}(\rho^a \otimes \rho_k^{a'})(U^{aa'})^\dagger$$
$$\Lambda_k^b(\rho^b) := \mathrm{tr}_{b'} U^{bb'}(\rho^b \otimes \rho_k^{b'})(U^{bb'})^\dagger.$$

By the convexity and monotonicity of concurrence, the desired result follows from

$$\begin{aligned} C(\Lambda^{ab}(\rho^{ab})) &= C(\sum_k v_k \Lambda_k^a \otimes \Lambda_k^b(\rho^{ab})) \\ &\leqslant \sum_k v_k C(\Lambda_k^a \otimes \Lambda_k^b(\rho^{ab})) \\ &\leqslant \sum_k v_k C(\rho^{ab}) = C(\rho^{ab}). \end{aligned}$$

From (iv), we readily see that if there is no entanglement in the initial state ρ^{ab}, then no entanglement can be created by the coupled channel Λ^{ab} if the joint environmental state $\rho^{a'b'}$ is also separable. On the other hand, if the channels Λ^a and Λ^b are entanglement breaking, then the final state $\Lambda^a \otimes \Lambda^b(\rho^{ab})$ after the action of the uncoupled channel is always separable even if the initial state ρ^{ab} is maximally entangled. But how about the final state $\Lambda^{ab}(\rho^{ab})$ after the action of the coupled channel? In sharp contrast, a remarkable phenomenon, as will be revealed in the subsequent calculation, is that $\Lambda^{ab}(\rho^{ab})$ may be entangled even if the environmental state $\rho^{a'b'}$ is separable! That is, even if the uncoupled channel $\Lambda^a \otimes \Lambda^b$ is entanglement breaking, and thus completely destroys all entanglement in the state ρ^{ab}, the coupled channel Λ^{ab} may preserve some entanglement in ρ^{ab} if the environments are correlated in a separable way. In this sense, the correlations in the environmental state $\rho^{a'b'}$ serve as a catalyst in preserving the entanglement in ρ^{ab}. The interesting point here is that if ρ^{ab} is not entangled, then $\rho^{a'b'}$ cannot be used for creating entanglement between

a and b. Only if there is already some entanglement in ρ^{ab}, can the correlations in $\rho^{a'b'}$ be useful in preserving entanglement between a and b.

3 An Illustrative Example

The above statement can be most directly and dramatically illustrated in two-qubit systems. Let the system state be the Werner state[29]

$$\rho^{ab} := (1-p)\frac{\mathbf{1}^{ab}}{4} + p|\Psi^{ab}\rangle\langle\Psi^{ab}| \tag{5}$$

with parameter $p \in [-\frac{1}{3}, 1]$. It is well known that ρ^{ab} is separable if $p \leqslant 1/3$ and is entangled if $p > 1/3$. Here $|\Psi^{ab}\rangle := \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ is the singlet state. Let the joint environmental state be another Werner state

$$\rho^{a'b'} := (1-p')\frac{\mathbf{1}^{a'b'}}{4} + p|\Psi^{a'b'}\rangle\langle\Psi^{a'b'}| \tag{6}$$

with parameter p'. For our illustrative purpose, we will parameterize the channels as $\{U^{aa'}, \mathbf{1}^{a'}\}$, $\{U^{bb'}, \mathbf{1}^{b'}\}$ and $\{U^{aa'} \otimes U^{bb'}, \rho^{a'b'}\}$ with $U^{aa'} = U^{bb'} = U$ given by

$$\begin{aligned} U &= e^{-i\sum_{k=1}^{3} c_k \sigma_k \otimes \sigma_k} \\ &= \begin{pmatrix} \bar{r}c_- & 0 & 0 & -i\bar{r}s_- \\ 0 & rc_+ & -irs_+ & 0 \\ 0 & -irs_+ & rc_+ & 0 \\ -i\bar{r}s_- & 0 & 0 & \bar{r}c_- \end{pmatrix}. \end{aligned} \tag{7}$$

Here σ_k are the Pauli spin matrices and c_k are real numbers satisfying $\pi/4 \geqslant c_1 \geqslant c_2 \geqslant |c_3|$, and

$$r := e^{ic_3}, \quad c_\pm := \cos(c_1 \pm c_2), \quad s_\pm := \sin(c_1 \pm c_2).$$

The matrix is with respect to the computational base $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. Noting that U is of a rather general form since any unitary operator acting on two-qubit space, up to local unitary equivalence, can be expressed as the above form, due to the Cartan decomposition theorem[30,31]. We need to evaluate

$$\begin{aligned} &\Lambda^{ab}(\rho^{ab}) \\ &= \mathrm{tr}_{a'b'}(U^{aa'} \otimes U^{bb'})(\rho^{ab} \otimes \rho^{a'b'})(U^{aa} \otimes U^{bb'})^\dagger \end{aligned} \tag{8}$$

with $U^{aa'} = U^{bb'} = U$. Noting that $U^{aa'}$ acts on aa' and $U^{bb'}$ acts on bb'. The explicit expression is given in the Appendix. For the particular case $c_1 = \pi/4, c_2 = c_3 = 0$, we have $c_- = c_+ = s_- = s_+ = 1/\sqrt{2}, r = 1$. By the formula in the Appendix,

$$\Lambda^{ab}(\rho^{ab}) = \frac{1}{4}\begin{pmatrix} 1+pp' & 0 & 0 & -p-pp' \\ 0 & 1-pp' & -p+pp' & 0 \\ 0 & -p+pp' & 1-pp' & 0 \\ -p-pp' & 0 & 0 & 1+pp' \end{pmatrix}.$$

First, we consider the case $p' = 0$ and thus the initial environmental state is a product state $\rho^{a'} \otimes \rho^{b'}$, we see readily that in this situation, the channels are actually uncoupled, and the final system state $\Lambda^{ab}(\rho^{ab}) = \Lambda^a \otimes \Lambda^b(\rho^{ab})$ is always separable for any p (even for $p = 1$ in which case the initial system state ρ^{ab} is maximally entangled). Thus the channels Λ^a and Λ^b are entanglement breaking in this situation.

Second, suppose that we correlate $\rho^{a'}$ and $\rho^{b'}$ in a Werner state with $p' \neq 0$. By the PPT criterion for two-qubit entanglement[32,33], we know that $\Lambda^{ab}(\rho^{ab})$ is entangled if and only if

$$p(1 + 2p') > 1.$$

In particular, if we take $p' = 1/3$ (which ensues that $\rho^{a'b'}$ is separable), then the state $\Lambda^{ab}(\rho^{ab})$ is entangled if $p > 3/5$, which stands in sharp contrast to the separability of $\Lambda^a \otimes \Lambda^b(\rho^{ab})$ for any p. Thus, here the *separable correlations* in the environmental state $\rho^{a'b'}$ help preserve some entanglement in the system state ρ^{ab}, which would be completely broken if the environmental correlations were absent.

The entanglement of $\Lambda^{ab}(\rho^{ab})$, as quantified by the concurrence $C(\cdot)$[23,28], can be readily evaluated as

$$C(\Lambda^{ab}(\rho^{ab})) = \max\left\{0, \frac{p(1 + 2p') - 1}{2}\right\}.$$

Clearly, if $p' = 1/3$, then $C(\Lambda^{ab}(\rho^{ab})) > 0$ for $p > 3/5$. However, $C(\Lambda^a \otimes \Lambda^b(\rho^{ab})) = 0$ for any p. In particular, if the initial state is maximally entangled (the Werner state with $p = 1$), then $C(\Lambda^{ab}(\rho^{ab})) = 1/3$, which should be compared with $C(\Lambda^a \otimes \Lambda^b(\rho^{ab})) = 0$.

It is interesting to check how much entanglement is lost due to the action of the coupled channel Λ^{ab}. For simplicity, we take $p' = 1/3$ and assume that $p > 3/5$, then the concurrence of the original Werner state ρ^{ab} and that of the final state $\Lambda^{ab}(\rho^{ab})$ is

$$C(\rho^{ab}) = \frac{3p - 1}{2}, \qquad C(\Lambda^{ab}(\rho^{ab})) = \frac{5p - 3}{6}$$

respectively. The concurrence decreasing is

$$C(\rho^{ab}) - C(\Lambda^{ab}(\rho^{ab})) = \frac{2}{3}p.$$

4 Conclusion

In summary, we have illustrated a systematic way for coupling two channels by use of environmental correlations through very simple examples. The fundamental properties of this channel coupling are investigated. A curious and remarkable effect of separable environmental correlations is revealed. This effect can be exploited to preserve certain entanglement, which otherwise would be completely broken if environments were not correlated. Although separable environmental correlations cannot create entanglement out of separable system states, they do contribute to preserving the existent entanglement. This corroborates the observation of separable correlations as a physical resource[13–19,34–37]. The channel coupling method for entanglement preservation provides an intuitive scheme for engineering environment, and may be applied to realistic situations in suppressing decoherence. To what extent experimental realization of this scheme is feasible is a further issue worthy investigation.

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Appendix. Here we present the analytical expression of $\Lambda^{ab}(\rho^{ab})$ defined by Eq. (8). For the Werner states ρ^{ab} and $\rho^{a'b'}$ given by Eqs. (5) and (6), $U^{aa'} = U^{bb'} = U$ given by Eq. (7), the final state

$$\Lambda^{ab}(\rho^{ab}) := \mathrm{tr}_{a'b'}(U^{ab} \otimes U^{a'b'})(\rho^{ab} \otimes \rho^{a'b'})(U^{ab} \otimes U^{a'b'})^{\dagger}$$

can be analytically evaluated as

$$\Lambda^{ab}(\rho^{ab}) = \frac{1}{4}\Big((1-p)(1-p')\mathbf{1}^{ab} + (1-p)p'\tau_1^{ab} \\ +p(1-p')\tau_2^{ab} + pp'\tau_3^{ab}\Big).$$

Here

$$\tau_1^{ab} = \begin{pmatrix} x_{11} & 0 & 0 & x_{14} \\ 0 & x_{22} & x_{23} & 0 \\ 0 & x_{32} & x_{33} & 0 \\ x_{41} & 0 & 0 & x_{44} \end{pmatrix}$$

$$\tau_2^{ab} = \begin{pmatrix} y_{11} & 0 & 0 & y_{14} \\ 0 & y_{22} & y_{23} & 0 \\ 0 & y_{32} & y_{33} & 0 \\ y_{41} & 0 & 0 & y_{44} \end{pmatrix}$$

$$\tau_3^{ab} = \begin{pmatrix} z_{11} & 0 & 0 & z_{14} \\ 0 & z_{22} & z_{23} & 0 \\ 0 & z_{32} & z_{33} & 0 \\ z_{41} & 0 & 0 & z_{44} \end{pmatrix}$$

with

$$x_{11} = x_{44} = (c_-^2 + s_+^2)(c_+^2 + s_-^2)$$

$$x_{22} = x_{33} = 2 - (c_-^2 + s_+^2)(c_+^2 + s_-^2)$$
$$x_{23} = x_{32} = \frac{1}{2}(\bar{r}^2 - r^2)^2(c_-^2 s_+^2 + c_+^2 s_-^2)$$
$$x_{14} = x_{41} = -(\bar{r}^2 - r^2)^2 c_- c_+ s_- s_+$$

$$y_{11} = y_{44} = (c_-^2 + c_+^2)(s_-^2 + s_+^2)$$
$$y_{22} = y_{33} = 2 - (c_-^2 + c_+^2)(s_-^2 + s_+^2)$$
$$y_{23} = y_{32} = -\frac{1}{2}(\bar{r}^2 + r^2)^2(c_-^2 c_+^2 + s_-^2 s_+^2)$$
$$y_{14} = y_{41} = -(\bar{r}^2 + r^2)^2 c_- c_+ s_- s_+$$

$$z_{11} = z_{44} = 4(c_-^2 s_-^2 + c_+^2 s_+^2)$$
$$z_{14} = z_{41} = -4(\bar{r}^4 + r^4) c_- c_+ s_- s_+$$
$$z_{22} = z_{33} = 2 - 4(c_-^2 s_-^2 + c_+^2 s_+^2)$$
$$z_{23} = z_{32} = -(\bar{r}^4 + r^4)(c_-^2 - s_-^2)(c_+^2 - s_+^2)$$