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Positive Aspects of Noise Effect on Quantum Correlations

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Abstract Quantum correlations, including entanglement, quantum discord, teleportation fidelity, etc., are resources for quantum information processing. Unavoidable quantum noise usually causes decreasing of quantum correlations, and thus affect the efficiency of quantum computation and communication. However, evidences show that proper quantum noise can increase quantum correlations under special conditions. This is because some quantum noise can rebuild the coherence of mixed quantum states. This article reviews the positive aspects of noise effect on quantum correlations, including collective noise, which is caused by several qubits interacting with a common reservoir, as well as individual noise, which is caused by each qubit locally coupled to its own reservoir.
Key words: quantum noise; quantum correlation; entanglement

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1 Introduction

Quantum correlations express the nonlocality between quantum particles. The debates on this spooky nonlocality was raised since early days of quantum mechanics. For experimental test of quantum nonlocality, Bell derived a set of inequalities which should be obeyed by any local theory[4]. Experimental results violet Bell's inequalities confirm that nonlocal states are physical reality[14]. Since then, the quantum entangled states, which can not be prepared locally, have attracted much attention. Recently, researches show that quantum states without entanglement can also possess quantum correlations which have no classical counterparts, including quantum discord, quantum deficit, etc.

The advantage of quantum information processing relies on the quantum correlation properties of quantum states. Quantum teleportation, which transports an unknown quantum state without transferring the physical carrier of the state, can only be achieved when the sender and receiver share entanglement[5]. Besides,

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quantum correlations beyond entanglement are proved to be the resource for several quantum protocols, such as quantum speed-up of quantum computation[11], remote state preparation[10], and quantum metrology[16], just to mention a few important examples.

Due to unavoidable interaction with environment, quantum states decohere, affecting the amount of quantum correlations. During the last two decades, various studies are devoted to the dynamics of quantum correlations under decoherence. The quantum correlations are usually decreased by quantum noises. Especially, entanglement may vanish in finite time, which is known as entanglement sudden death[13]. However, proper noises can contribute to quantum correlations. Quantum entanglement can even be created by collective decoherence under some conditions[6]. Further, any Markovian collective decoherence can create quantum discord for some initial states[20]. Local noises can never increase quantum entanglement from definition. However, the ability of some entangled state in teleportation can be enhanced by local operations[2,3,35]. Besides, quantum correlations beyond entanglement can be created and increased by proper local noises[8,9,20,21,26,32]. All these researches show that well-controlled quantum noises can have positive effects on quantum correlations. In this paper, we will review recent works about the positive aspect of quantum noise effect on quantum correlations.

This paper is organized as follows. In Sec. 2 and 3, we will review the definition of quantum correlations, including quantum entanglement, quantum discord, etc., and the origin and description of quantum noises. Sec. 4 is devoted to the positive effect of collective quantum noises on quantum entanglement and discord. The effect of local quantum noise on quantum correlations is reviewed in Sec. 5.

2 Quantum Correlations

Quantum correlations originate from the superposition of quantum states, which is one of the basic hypotheses of quantum mechanics. For a classical bit, it can be either 0 or 1. For a quantum bit, it can be in state $|0\rangle$ and state $|1\rangle$ at the same time. The quantum state of the qubit can be described as the wave function $|\psi\rangle = \sqrt{p}|0\rangle + \sqrt{1-p}|1\rangle$, where p can be understood as the probability of the qubit in state $|0\rangle$. The superposition of quantum states have been observed in two-split experiment.

When we consider two qubits, with half probability both in state $|0\rangle$ and half probability both in state $|1\rangle$, the state can be written as $|\Psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$, from the superposition of quantum states. For the state $|\Psi\rangle$, the two qubits are correlated, in the sense that if we measure one of the qubits and obtain for example $|0\rangle$, we know for sure that the other qubit is in state $|0\rangle$ too. Moreover, when we measure the qubits on other basis, e.g., $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$, correlated measurement results would still be obtained. On the other hand, if the two qubits are classically correlated, the state is just a probabilistic sum of the two cases $\rho = (|00\rangle\langle 00| + |11\rangle\langle 11|)/2$. The measurement results of the two qubits are correlated for measurement basis $|0\rangle$ and $|1\rangle$, but the results are totally uncorrelated for measurement basis $|+\rangle$ and $|-\rangle$. Obviously, the two qubit state $|\Psi\rangle$ possesses quantum nonlocality beyond classical correlation. It is an entangled state.

2.1 Entanglement

Quantum entanglement is the basic problem of quantum mechanics, and the resource for quantum information processing. During the past two decades, quantum entanglement has drawn much interest, focusing on its definition, measure, dynamics under decoherence, etc.

An entangled state is defined as a quantum state of two or more quantum particles, which can not be prepared via local operation and classical communication (LOCC). Mathematically, the density matrix of an entangled state can not be written as the sum of products of density matrices of individual particles as Ref. [31]

$$\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B \otimes \cdots , \tag{1}$$

where $A, B, \cdots$ are labels of quantum particles and p_i's are probabilities satisfying $\sum_i p_i = 1$.

There are various measures of quantum entanglement, all of which should satisfy the following three conditions[18].

(a) For separable states which can be prepared via LOCC, the quantum entanglement measure E vanishes, while $E = 1$ for maximally entangled state.

(b) LOCC can not increase the amount of quantum entanglement.

(c) Local unitary operator can not change the amount of quantum entanglement.

Here we briefly review some of the entanglement measures which are commonly used.

Entanglement entropy[1] is used to measure the entanglement contained in a pure state $|\Psi_{AB}\rangle$ of a bipartite system. It is defined as the von Neumann entropy of the subsystem A or B

$$E_S(|\Psi_{AB}\rangle) = S_A(|\Psi_{AB}\rangle) = -\mathrm{Tr}(\rho_A \log_2 \rho_A) = S_B(|\Psi_{AB}\rangle). \tag{2}$$

Here ρ_A is the density matrix of the subsystem A. It should be mentioned that the entanglement entropy is the only entanglement measure for bipartite pure states that satisfy the conditions (a-c), in the sense that all the other entanglement measures that satisfy the three conditions should be equivalent to entanglement entropy in the case of bipartite pure states.

Entanglement of formation (EoF)[17,34] is defined as the minimum pairs of singlet states that is required on average to prepare a two qubit state ρ. For a two-qubit pure state, the entanglement of formation E_F equals to the entanglement entropy. For a two-qubit mixed state ρ, entanglement of formation equals to the average EoF of the optimal pure state decomposition $\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$, which can be written as

$$E_F(\rho) = \inf_{\{p_i, |\Psi_i\rangle\}} \left\{ \sum_i p_i E_F(|\Psi_i\rangle\langle\Psi_i|) \right\} . \tag{3}$$

The analytical expression for E_F is derived and related to the so-called "concurrence" as

$$E_F(\rho) = \mathrm{E}(C(\rho)), \tag{4}$$

where $\mathrm{E}(x) = H\left(\frac{1}{2}(1+\sqrt{1-x^2})\right)$, $H(x) = -x\log x - (1-x)\log(1-x)$ is the Shannon entropy of the random variable x, and

$$C(\rho) = \max\{0, \lambda_{\max} - \mathrm{Tr}R\} \tag{5}$$

is called concurrence. Here $\lambda_{\max}$ is the maximal eigenvalue of the matrix $R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ and $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho(\sigma_y \otimes \sigma_y)$.

Negativity[33] is the quantification of negative partial transpose (NPT). NPT is an entanglement criterion, which is stated as follows. For a separable state in form of Eq. (1), when a partial transposing is taken on the subsystem A, the eigenvalues of the resulting matrix $\rho^{\mathrm{PT}} \equiv \sum_i p_i(\rho_i^A)^{\mathrm{T}} \otimes \rho_i^B \otimes \cdots$ should be non-negative. Therefore, a negative eigenvalue of matrix ρ^{PT} means that the state can not be written in form of Eq. (1) so it is an entangled state. Negativity is defined as

$$N(\rho) = \max\left\{0, -\sum_k \lambda_k^-\right\}, \tag{6}$$

where λ_k^- are the negative eigenvalues of matrix ρ^{PT}.

2.2 Quantum correlations beyond entanglement

Recent researches show that quantum states with vanishing entanglement can possess correlations which can not be revealed by local measurements and classical communication, and that such correlations can be useful in the speed up of quantum computation. The quantum correlation beyond entanglement has become a research interest of quantum information theory, see Ref. [27] for a nice review. Here we briefly introduce the definition and properties of quantum discord, and other measures of quantum correlations beyond entanglement.

The definition of quantum discord is raised when quantum measurement is considered[29]. In the process of quantum measurement, the correlation is built between the system S and the detector A, and then one has access to the information of S by reading the detector A. In classical information theory, the system S and the detector A are described by random series. The mutual information of the two random series is defined as

$$\mathcal{C}(S:A) = H(S) - H(S|A). \tag{7}$$

Here $H(X) = -\sum_x p(X=x)\log_2 p(X=x)$ is the Shannon entropy, which stands for the information contained in X $(X = S, A)$, and $H(S|A) = \sum_a p(A=a)H(S|A=a)$ is the unknown information of S after reading the measurement results of A. Therefore, the mutual information describes the information in S which is revealed by reading the detector A. From Bayes theroy, $H(S|A) = H(S,A) - H(A)$, where $H(S,A)$ is the total information contained in system $S-A$. Thus, one obtains another equivalent definition of mutual information

$$\mathcal{I}(S:A) = H(S) + H(A) - H(S,A). \tag{8}$$

When we consider quantum measurement, where the system S, detector A and the composite system $S-A$ are described by their density matrices ρ_S, ρ_A, and

ρ_{SA}. The corresponding information entropy is generalized to von Neumann entropy $\mathcal{S}(\rho_X) = -\mathrm{Tr}(\rho_X \log_2 \rho_X)$. Thus the quantum generalization of Eq. (8) is

$$\mathcal{C}(\rho_{SA}) = \max_{\{\Pi_k^A\}}[\mathcal{S}(\rho_S) - \sum_k p_k \mathcal{S}(\rho_{S|\Pi_k^A})]. \tag{9}$$

Here $\{\Pi_k^A\}$ is the orthogonal projector measurement on A, $p_k = \mathrm{Tr}(\rho_{SA}\Pi_k^A)$ is the probability to get the result a_k, and $\rho_{S|\Pi_k^A}$ is the density matrix of S when the measurement result is a_k. The physical quantity in Eq. (9) is called classical correlation, which describes the information of S that can be obtained by reading the detector A. On the other hand, the physical generalization of Eq. (8)

$$\mathcal{I}(\rho_{SA}) = \mathcal{S}(\rho_S) + \mathcal{S}(\rho_A) - \mathcal{S}(\rho_{SA}), \tag{10}$$

is called quantum mutual information, which describes the total correlation between S and A. Eqs. (9) and (10) are not equivalent, and the difference between them is called quantum discord

$$\mathcal{D}(\rho_{SA}) = \mathcal{I}(\rho_{SA}) - \mathcal{C}(\rho_{SA}). \tag{11}$$

Quantum discord describes the quantumness of the correlation between S and A, and satisfies the following conditions.

(a) Quantum discord vanishes for quantum-classical states. The set of all quantum-classical states $\mathcal{C}_0$ can be written as Ref. [12]

$$\mathcal{C}_0 = \left\{\rho|\rho = \sum_i q_i \rho_i^S \otimes \Pi_{\alpha_i}^A\right\}, \tag{12}$$

where $\{\Pi_{\alpha_i}^A = |\alpha_i\rangle\langle\alpha_i|\}$ are a set of orthogonal basis of part A.

(b) Quantum discord can not be changed by local unitary operators $\mathcal{D}(U_S \otimes U_A \rho_{SA} U_S^\dagger \otimes U_A^\dagger) = \mathcal{D}(\rho_{SA})$, where U_S and U_A are arbitrary local unitary operators on S and A.

(c) Quantum discord defined on A can not be increased by the local operations on S $\mathcal{D}(\Lambda_S \otimes I_A(\rho_{SA})) \leqslant \mathcal{D}(\rho_{SA})$[22].

These three conditions are satisfied by most of the quantum correlation measures beyond entanglement. The distance-based measure of quantum correlation $Q_D(\rho) = \min_{\sigma\in\mathcal{C}_0} D(\rho,\sigma)$, where the state distance satisfies that D does not increase under any quantum operation. Trace-norm distance $D_1 = \mathrm{Tr}|\rho - \sigma|/2$ with $|\hat{O}| = \sqrt{\hat{O}^\dagger\hat{O}}$ and relative entropy $S(\rho \parallel \sigma) = \mathrm{Tr}[\rho(\log_2 \rho - \log_2 \sigma)]$ are examples satisfying this property. One-way quantum deficit

$$\Delta_{B|A}^{\leftarrow} = \min_{\{\Pi_A\}} S(\rho_{\Pi_i^A B}) - S(\rho), \tag{13}$$

is in fact the minimum relative entropy to classical-quantum states, and thus belongs to this class of quantum correlation measure. Here we briefly prove that Q_D satisfies condition (c). Suppose the closest classical-quantum state to ρ is labeled as σ, then we have $Q_D(\rho) = D(\rho,\sigma) \geqslant D(\Lambda_B(\rho),\Lambda_B(\sigma)) \geqslant Q_D(\Lambda_B(\rho))$. The last inequation holds because $\Lambda_B(\sigma)$ is still a quantum-classical state, but may not be the closest one to $\Lambda_B(\rho)$. It should be noticed that geometric quantum discord does not satisfy condition (c).

3 Quantum Noise

In quantum communication, quantum particles are sent through quantum channels. Noisy quantum channels affect the quantum states of particles as $\rho \rightarrow \Lambda(\rho)$.

Mathematically, the quantum channel $\Lambda(\cdot)$ can be presented by Kraus operators[28]. Quantum noise is caused by interaction between system and reservoir. Suppose ρ and $\rho_{\rm env} = |e_0\rangle\langle e_0|$ are the initial states of system and the reservoir respectively, and the unitary operator U describes the interaction between system and reservoir. Then the output state of the system is

$$\begin{aligned} \Lambda(\rho) &= \mathrm{Tr}_{\rm env}\left[U(\rho \otimes \rho_{\rm env})U^\dagger\right] \\ &= \sum_k \langle e_k|U(\rho \otimes |e_0\rangle\langle e_0|)U^\dagger|e_k\rangle \\ &= \sum_k \mathbb{E}_k \rho \mathbb{E}_k^\dagger, \end{aligned} \tag{14}$$

where $\mathbb{E}_k \equiv \langle e_k|U|e_0\rangle$ are called Kraus operators, which contains all the information of the noisy quantum channel $\Lambda(\cdot)$. Kraus operators possess the following properties.

(a) $\sum_k \mathbb{E}_k^\dagger \mathbb{E}_k = \mathbb{I}$, which can be obtained by the trace-preserving property of quantum channel $\Lambda(\cdot)$.

(b) For the same quantum channel, there are infinite Kraus operators representations. Two sets of Kraus operators $\{\mathbb{E}_1, \mathbb{E}_2, \cdots, \mathbb{E}_m\}$ and $\{\mathbb{F}_1, \mathbb{F}_2, \cdots, \mathbb{F}_m\}$ are equivalent if and only if they are related as $\mathbb{E}_i = \sum_j u_{ij}\mathbb{F}_j$, where u_{ij} are the elements of an $m \times m$ unitary matrix.

Kraus operator representation gives general description of a quantum channel, but it does not give the dynamics of the quantum state. In order to study the dynamics of quantum system under noise, master equations are introduced for Markovian process. Here we briefly derive the master equations following the lines in Ref. [15].

The total Hamiltonian of the system S and the reservoir B is

$$H = H_S + H_B + H_{\rm Int}, \tag{15}$$

where H_S, H_B, and $H_{\rm Int}$ are respectively Hamiltonians for the system, reservoir, and the interaction between them. In Schrödinger representation, the dynamics of the total state is described by $\dot{\rho}_{\rm tot} = -i[H, \rho_{\rm tot}]/\hbar$, and the density matrix of the system at time t is $\hat{\rho}(t) = \mathrm{Tr}_B(\rho_{\rm tot}(t))$. When transformed to interaction representation, the total state of system and reservoir becomes $\rho_I(t) = \exp[i(H_S + H_B)t/\hbar]\ \rho_{\rm tot}(t) \exp[-i(H_S + H_B)t/\hbar]$. The density matrix of system is $\hat{\rho}(t) = \exp[-itH_S/\hbar]\rho(t)\ \exp[itH_S/\hbar]$, where

$$\rho(t) \equiv \mathrm{Tr}_B[\rho_I(t)] \tag{16}$$

is the reduced density matrix of system in interaction representation. Following approximations are made for deriving master equations. (a) The system and environment are initially uncorrelated $\rho_{\rm tot} = \hat{\rho}(0) \otimes \rho_B$. Furthermore, the reservoir is large so that it is not affected by the system. (b) The system and reservoir are weakly coupled and thus at any time t we have $\rho_{\mathrm{tot}(t)} \approx \hat{\rho}(t) \otimes \rho_B$. (c) Markovian approximation: the reservoir is memoryless, and the dynamics of system only depend on the current state. The general form of master equations in the interaction representation is then

$$\dot{\rho}(t) = -\frac{1}{\hbar^2}\int_0^\infty d\tau \mathrm{Tr}_B\{[H_{\mathrm{Int}}(t), [H_{\mathrm{Int}}(t-\tau), \rho(t) \otimes \rho_B]]\}. \tag{17}$$

In the case of linear interaction, the interaction Hamiltonian in Eq. (15) can be written as the general form

$$H_{\mathrm{Int}} = \hbar \sum_m (X_m^+ \Gamma_m + X_m^- \Gamma_m^\dagger), \tag{18}$$

Here $X_m^\pm$ are the eigen-operators of the system Hamiltonian satisfying $[H_S, X_m^\pm] = \pm\hbar\omega_m X_m^\pm$, and Γ_n are the reservoir operators. Under rotating wave approximation, the master equation can be written as

$$\begin{aligned}\dot{\rho}(t) = &-i\sum_m [\delta_m X_m^+ X_m^- + \epsilon_m X_m^- X_m^+, \rho] \\ &+\frac{1}{2}\sum_m K_m(2X_m^-\rho X_m^+ - X_m^+ X_m^- \rho - \rho X_m^+ X_m^-) \\ &+\frac{1}{2}\sum_m G_m(2X_m^+\rho X_m^- - X_m^- X_m^+ \rho - \rho X_m^- X_m^+),\end{aligned} \tag{19}$$

where the coefficients are defined as $\int_0^\infty d\tau \mathrm{e}^{i\omega_m\tau}\mathrm{Tr}_B[\Gamma_m(\tau)\Gamma_m^\dagger(0)\rho_B] \equiv \frac{1}{2}K_m + i\delta_m$, $\int_0^\infty d\tau \mathrm{e}^{-i\omega_m\tau}\mathrm{Tr}_B[\Gamma_m(0)\Gamma_m^\dagger(\tau)\rho_B] \equiv \frac{1}{2}K_m - i\delta_m$, $\int_0^\infty d\tau \mathrm{e}^{i\omega_m\tau}\mathrm{Tr}_B[\Gamma_m^\dagger(\tau)\Gamma_m(0)\rho_B] \equiv \frac{1}{2}G_m + i\epsilon_m$, $\int_0^\infty d\tau \mathrm{e}^{-i\omega_m\tau}\mathrm{Tr}_B[\Gamma_m^\dagger(0)\Gamma_m(\tau)\rho_B] \equiv \frac{1}{2}G_m - i\epsilon_m$. The first two terms of Eq. (19) are Lamb shift and Stark shift respectively, the last two lines are the decoherence caused by reservoir.

4 Effect of Quantum Collective Noise

When quantum systems are close to each other, they interact with the same reservoir, resulting in collective decoherence. Mediated by the reservoir, the correlation between quantum systems can be built.

4.1 Creation of quantum entanglement

Considered a simple model with two two-level atoms coupled to a common thermostat at zero temperature[24]. The two atoms are located within the radiation wavelength but not close enough to evoke the Jaynes-Cummings coupling. With a substantial probability, the photon emitted by one atom will be absorbed by the other. This mechanism may build entanglement between two atoms. In the Markovian approximation, the dynamics of the two atoms is described by the Master equation

$$\frac{d\rho}{dt} = L\rho, \tag{20}$$

with the Lindblad operator L

$$L\rho = \frac{\gamma_0}{2}\left[2\sigma_-^A\rho\sigma_+^A + 2\sigma_-^B\rho\sigma_+^B - (\sigma_+^A\sigma_-^A + \sigma_+^B\sigma_-^B)\rho - \rho(\sigma_+^A\sigma_-^A + \sigma_+^B\sigma_-^B)\right]$$

$$+\frac{\gamma}{2}\left[2\sigma_-^A\rho\sigma_+^B + 2\sigma_-^B\rho\sigma_+^A - (\sigma_+^A\sigma_-^B + \sigma_+^B\sigma_-^A)\rho - \rho - (\sigma_+^A\sigma_-^B + \sigma_+^B\sigma_-^A)\right]. \quad (21)$$

Here $\sigma_+^{A(B)} = |1\rangle_{A(B)}\langle 0|$ and $\sigma_-^{A(B)} = |0\rangle_{A(B)}\langle 1|$ are the raising and lowering operators of atom $A(B)$, γ_0 is the single atom spontaneous emission rate, and γ is the photon exchange rate, which increases as the two-atom distance decreases and reaches γ_0 in the limit of zero distance. By analytically solving the Eq. (20) with $\gamma \approx \gamma_0$, the concurrence of asymptotic state equals to Ref. [24]

$$C(\rho_{as}) = \frac{1}{2}|\rho_{22} + \rho_{33} - 2\text{Re}\rho_{23}|, \quad (22)$$

where ρ_{jk} are the matrix elements of the initial state on basis $\hat{e}_1 = |11\rangle, \hat{e}_2 = |10\rangle, \hat{e}_3 = |01\rangle, \hat{e}_4 = |00\rangle$. In the case with $\gamma < \gamma_0$, the single atom spontaneous emission is in charge of the evolution and the asymptotic state is $|00\rangle$ for any input state. However, entanglement can still be created at finite time.

Consider two qubits interacting with a heat bath in thermal equilibrium at temperature T. The two qubits do not interact with each other, so the Hamiltonian is $H_{AB} = H_A + H_B$. By assuming further that the states $|0\rangle$ and $|1\rangle$ are energy eigenstates with degenerate energies for both qubits, we have $H_{AB} = 0$ up to a constant. The heat bath Hamiltonian is composed of a collection of N harmonic oscillators $H_{\text{bath}} = \sum_{n=1}^{N}(\frac{1}{2m}p_i^2 + \frac{1}{2}m\omega_i^2 q_i^2)$. The interaction between qubits and heat bath is $H_{\text{int}} = (S^A + S^B)B$ with $B = \sum_k g_k q_k$, g_k the coupling constant, and S^A, S^B the "coupling agents" acting on the two qubits respectively. They strictly solve the evolution of the quantum entanglement (measured by negativity) between the two qubits for the situation with initial state[7]

$$W(0) = |\phi^A\rangle\langle\phi^A| \otimes |\phi^B\rangle\langle\phi^B| \otimes \frac{1}{Z}\exp(-H_{\text{bath}}/k_B T), \quad (23)$$

where $Z = \text{Tr}_{\text{bath}}\exp(-H_{\text{bath}}/k_B T)$ with k_B being the Boltzmann's constant, $|\phi^A\rangle = (|0\rangle_A - |1\rangle_A)/\sqrt{2}$, and $|\phi^B\rangle = (|0\rangle_B - |1\rangle_B)/\sqrt{2}$. Here $|0\rangle_{A(B)}$ and $|1\rangle_{A(B)}$ are eigenstates of $S^A(S^B)$ with eigenvalues 0 and 1. The calculation results show that the entanglement vanishes at time $t = 0$ and become strictly positive at finite times. It reaches a maximal value $\frac{1}{2}$ and vanishes asymptotically.

A general study on the entanglement between two noninteracting two-level system induced by Markovian dissipative dynamics was given in Ref. [6]. They consider the general form of master equation in form of Eq. (20) with Lindblad operator

$$\begin{aligned} L[\rho] = \sum_{i,j=1}^{3}\Bigg(&A_{ij}\left[(\sigma_i\otimes I)\rho(\sigma_j\otimes I) - \frac{1}{2}\{(\sigma_j\sigma_i\otimes I),\rho\}\right] \\ &+C_{ij}\left[(I\otimes\sigma_i)\rho(I\otimes\sigma_j) - \frac{1}{2}\{(I\otimes\sigma_j\sigma_i),\rho\}\right] \\ &+B_{ij}\left[(\sigma_i\otimes I)\rho(I\otimes\sigma_j) - \frac{1}{2}\{(\sigma_i\otimes\sigma_j),\rho\}\right] \\ &+B_{ij}^*\left[(I\otimes\sigma_j)\rho(\sigma_i\otimes I) - \frac{1}{2}\{(\sigma_i\otimes\sigma_j),\rho\}\right]\Bigg), \end{aligned} \quad (24)$$

where σ_i are Pauli matrices. The first two contributions are individual dissipative terms. The last two pieces represent the way in which the noise may correlate the two qubits. It is proved that entanglement can be created between the two qubits via dynamics described by Eq. (24) if and only if

$$\langle u|\hat{A}|u\rangle\langle v|\hat{C}^{\mathrm{T}}|v\rangle < |\langle u|\mathrm{Re}(\hat{B})|v\rangle|^2. \tag{25}$$

Here $\hat{A}$, $\hat{C}$ and $\hat{B}$ are 3×3 matrices whose matrix elements are the dissipative rate in Eq. (24), $|u\rangle$ and $|v\rangle$ are three-component vectors depending on the input product two-qubit state.

4.2 Creation of quantum correlation beyond entanglement

From Eq. (25), entanglement can be created only when the correlation effect caused by the collective noise surpasses the individual dissipative effect. It is an interesting question whether a coefficient matrix B which is nonzero but does not satisfy Eq. (25) can give rise to some quantum correlation between the two qubits. We give a positive answer to this question and prove that a nonzero $\hat{B}$ is necessary and sufficient for the creation of quantum discord[20]. We consider two quantum systems (qudits) with dimension d_A and d_B respectively, and the dynamics is described by Eq. (20) with

$$L[\rho(t)] = L_A[\rho(t)] + L_B[\rho(t)] + L_{AB}[\rho(t)], \tag{26}$$

where $L_A[\rho(t)] = \sum_{i,j=1}^{d_A^2-1}\hat{A}_{ij}[(\mathbb{F}_i\otimes\mathbb{I})\rho(t)(\mathbb{F}_j\otimes\mathbb{I}) - \{(\mathbb{F}_j\mathbb{F}_i\otimes\mathbb{I}),\rho\}/2]$ and $L_B[\rho(t)] = \sum_{i,j=1}^{d_B^2-1}\hat{C}_{ij}[(\mathbb{I}\otimes\mathbb{G}_i)\rho(t)(\mathbb{I}\otimes\mathbb{G}_j) - \{(\mathbb{I}\otimes\mathbb{G}_j\mathbb{G}_i),\rho\}/2]$ are dissipative terms individually affecting qudits A and B, whereas

$$L_{AB}[\rho(t)] = \sum_{i=1}^{d_A^2-1}\sum_{j=1}^{d_B^2-1}\left(\hat{B}_{ij}\left[(\mathbb{F}_i\otimes\mathbb{I})\rho(t)(\mathbb{I}\otimes\mathbb{G}_j) - \frac{1}{2}\{(\mathbb{F}_i\otimes\mathbb{G}_j),\rho\}\right] + \mathrm{H.c.}\right). \tag{27}$$

Here $\{\mathbb{F}_i\}_{i=0}^{d_A^2-1}(\mathbb{F}_0=\mathbb{I})$ $[\{\mathbb{G}_i\}_{i=0}^{d_B^2-1}(\mathbb{G}_0=\mathbb{I})]$ constitutes a complete basis for the vector space of bounded operators acting on the Hilbert space $\mathcal{H}_A$ ($\mathcal{H}_B$). The $(d_A^2-1)\times(d_A^2-1)$ coefficient matrix $\mathbb{A}=\mathbb{A}^\dagger$ and the $(d_B^2-1)\times(d_B^2-1)$ coefficient matrix $\mathbb{C}=\mathbb{C}^\dagger$ describe the individual decoherence acting on qudits A and B respectively, and the $(d_A^2-1)\times(d_B^2-1)$ coefficient matrix $\mathbb{B}$ corresponds to the correlation effect caused by the collective decoherence. Our proof is based on a criterion for nonzero quantum discord. If a state ϱ_{AB} has vanishing quantum discord on B, then

$$[\varrho_{AB}, \mathbb{I}_A\otimes\varrho_B] = 0, \tag{28}$$

where $\varrho_B = \mathrm{Tr}_A(\varrho_{AB})$ is the reduced density matrix for qudit B. We proved that, for any Markovian-dynamics described by Eq. (26) with nonzero $\hat{B}$, one can always find an initial product state $\rho(0)=\rho_A\otimes\rho_B$, whose corresponding dynamical state $\rho(t)$ violate Eq. (28). It means that a Markovian dissipative quantum channel can generate quantum discord from some bipartite product states if and only if it cannot be reduced to individual decoherence channels independently acting on each qudit. This builds a tight connection between quantum discord and collective decoherence.

5 Effect of Local Quantum Noise

5.1 Local increase of teleportation fidelity

Quantum teleportation, a protocol that transports an unknown quantum state from Alice to Bob without transferring the physical carrier of the state, is one of the most fantastic communication task enabled by quantum theory. In the ideal case, the sender Alice and receiver Bob share a pair of singlet state as follows

$$|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle), \tag{29}$$

and Alice wants to transport a single qubit state $|\phi\rangle_C = a|0\rangle + b|1\rangle$ without transferring the carrier qubit C. The initial state of the three qubits A, B, and C is

$$\begin{aligned}|\Xi\rangle_{CAB} &= |\phi\rangle_C \otimes |\Psi^-\rangle_{AB} \\ &= \frac{1}{2}[-|\Psi^-\rangle_{CA}|\phi\rangle_B - |\Psi^+\rangle_{CA}(\sigma_z|\phi\rangle_B) \\ &\quad +|\Phi^-\rangle_{CA}(\sigma_x|\phi\rangle_B) - |\Phi^+\rangle_{CA}(i\sigma_y|\phi\rangle_B)].\end{aligned} \tag{30}$$

Here $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ and $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ are four Bell states, and $\sigma_{x,y,z}$ are Pauli matrices. Therefore, when Alice measure her qubits C and A on the basis of Bell states, the quantum state of qubit B collapse to a known pure state which relates to the state $|\phi\rangle$ by a unitary operator. Bob then can recover the teleported state $|\phi\rangle$ on his qubit B by making a unitary operator according to Alice's measurement result.

When the shared entangled state is not a perfect maximally entangled state, the teleported state ρ_{out} will differ from the original state $|\phi\rangle$. The quality of the teleportation is measured by the average teleportation fidelity

$$f = \int d\phi \langle\phi|\rho_{out}|\phi\rangle. \tag{31}$$

For the standard teleportation protocol, the optimal teleportation fidelity depends on the previously shared entangled state ϱ_{AB} as $f_{\max} = (F_{\max}d + 1)/(d + 1)$, where d is the dimension of the qudit to be teleported, and $F_{\max} = \max_{|\Psi\rangle \in \mathrm{ME}} \langle\Psi|\varrho|\Psi\rangle$ is the maximally entangled fraction (ME means the maximally entangled state). Therefore, increase of $F_{\max}$ means the improvement of quantum teleportation.

Reference [2] gave the first example where local operation gives rise to an increase of $F_{\max}$. They focus on a family of two-qubit states

$$\varrho = \begin{pmatrix} \varrho_{11} & 0 & 0 & \varrho_{14} \\ 0 & \varrho_{22} & -p_{23} & 0 \\ 0 & -p_{23} & \varrho_{33} & 0 \\ \varrho_{14} & 0 & 0 & \varrho_{44} \end{pmatrix}, \tag{32}$$

where $p_{23} \geqslant (1 - \varrho_{22} - \varrho_{33})/2$ in order to make sure that the associated teleportation fidelity surpasses the classical limit. A local amplitude damping acts on Bob's qubit whose Kraus operators are

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{p} \end{pmatrix}, E_2 = \begin{pmatrix} 0 & \sqrt{1-p} \\ 0 & 0 \end{pmatrix}. \tag{33}$$

After some direct calculations, they show that the maximally entangled fraction is changed by

$$\begin{aligned}\Delta F &= F(\tilde{\varrho}) - F(\varrho)\\ &= (1-\sqrt{1-p})\left[\frac{1+\sqrt{1-p}}{2}(\varrho_{44}-\varrho_{22}) - p_{23}\right].\end{aligned} \tag{34}$$

For states ϱ in form of Eq. (32) satisfying $[\varrho_{44}-\varrho_{22}] - p_{23} \geqslant 0$, the teleportation fidelity can be improved by local amplitude damping on Bob's qubit.

For the case of two-qubit teleportation where four-qubit entangled states are shared between Alice and Bob as the teleportation channel states, the teleportation fidelity can also be increased by the local noises acting on two of the four-qubit states. Reference [3] considered a class of four-qubit entangled states shared between Alice and Bob. These states are obtained from maximally entangled four-qubit states where Bob's two qubits go through a pair of so-called time-correlated amplitude damping channels. For these states, when a pair of time-correlatedd amplitude damping channels also act on Alice's two qubits, the teleportation fidelity associated with the four-qubit states is increased. They also show that in this process, the entanglement of the teleported state is decreased while the quantum discord is increased.

In Ref. [19], authors considered another class of four-qubit states $\varrho_{1234}^{CAD} \equiv I_{12} \otimes \varepsilon_{34}^{CAD}(\varrho_{1234})$, which are obtained when Alice prepares a four-qubit state ϱ_{1234} with maximal entanglement between qubits $1,2$ and qubits $3,4$, and then transferred qubits to Bob through a collective amplitude damping channel ε_{34}^{CAD}. For states ϱ_{1234}^{CAD}, their ability to teleport a two-qubit state can be enhanced by collective amplitude damping channel on Alice's qubits 1 and 2. Moreover the entanglement of the teleported state can also be increased.

5.2 Local creation and increase of quantum discord

5.2.1. An example

Quantum discord exists for some separable states, and thus can be created and increased by local operations and classical communications. Actually, local operations alone can create quantum discord. For example, consider a two-qubit state $\rho_{AB} = \sum_{i=0}^{1} p_i\rho_i^A \otimes |\phi_i\rangle_B\langle\phi_i|$, where $|\phi_{0,1}\rangle_B = |\pm\rangle \equiv \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ and $\rho_{0,1}^A$ are two different density matrices of qubit A. Obviously, the state ρ_{AB} has zero quantum discord on B. Now amplitude damping channel is applied on qubit B, and the two-qubit state becomes

$$\rho'_{AB} = p_0\rho_0^A \otimes \begin{pmatrix} \frac{1+p}{2} & \frac{\sqrt{1-p}}{2} \\ \frac{\sqrt{1-p}}{2} & \frac{1+p}{2} \end{pmatrix}_B + p_1\rho_1^A \otimes \begin{pmatrix} \frac{1+p}{2} & -\frac{\sqrt{1-p}}{2} \\ -\frac{\sqrt{1-p}}{2} & \frac{1+p}{2} \end{pmatrix}_B. \tag{35}$$

Here we use Eq. (28) to check the quantum discord contained in ρ'_{AB}, and direct calculations lead to

$$[\rho'_{AB}, I_A \otimes \rho'_B] = p_0p_1(\rho_0^A - \rho_1^A) \otimes p\sqrt{1-p}\sigma_x^B, \tag{36}$$

which does not vanish for $0 < p < 1$. Thus, we conclude that amplitude damping acting locally on qubit B of the quantum-classical state ρ_{AB} can induce quantum discord between qubit A and qubit B and that the induced discord does not vanish in finite time.

5.2.2. Conditions on local creation of quantum correlation

An interesting result is that any separable state with positive quantum discord is a "shadow" of a classical state in a larger Hilbert space[25]. Precisely, when a bipartite state ρ on Hilbert space $\mathcal{H}^A \otimes \mathcal{H}^B$ is separable, then it can be obtained from a classical-classical state σ on $(\mathcal{K}^A \otimes \mathcal{H}^A) \otimes (\mathcal{K}^B \otimes \mathcal{H}^B)$ by tracing out the auxiliary Hilbert space $\mathcal{K}^A$ and $\mathcal{K}^B$: $\rho = \mathrm{Tr}_{\mathcal{K}^A \otimes \mathcal{K}^B} \sigma$.

For local channels which preserve the dimension of Hilbert space, the necessary and sufficient condition on local creation of quantum correlation has been derived for both the qubit case[32] and the quantum systems with higher dimensions[21].

Generally, the criterion for whether a local quantum channel is able to create quantum correlation is given by the following theorem[21]. *A local quantum channel* Λ *can not create quantum correlations if and only if it is a commutativity-preserving channel which is defined as*

$$[\Lambda(\xi_1), \Lambda(\xi_2)] = 0, \forall [\xi_1, \xi_2] = 0. \tag{37}$$

Proof of the theorem goes as follows. Any separable state can be written as

$$\xi_{AB} = \sum_i p_i \xi_i^A \otimes \xi_i^B, \tag{38}$$

where ξ_i^A are linearly independent density matrices of qudit A. We will first prove that ξ_{AB} is a quantum-classical state if and only if

$$[\xi_i^B, \xi_j^B] = 0, \ \forall i, j. \tag{39}$$

For proving the "only if" part, we notice that for any quantum-classical state, there exists an orthogonal measurement basis $\Pi_B^{\alpha_i}$ that does not affect the state. Therefore,

$$\sum_i p_i \xi_i^A \otimes (\xi_i^B - \Pi_B^{\alpha_j} \xi_i^B \Pi_B^{\alpha_j}) = 0. \tag{40}$$

Because ξ_i^A are linearly independent, ξ_i^B is diagonal on $\{\Pi_B^{\alpha_j}\}$ and thus satisfies Eq. (39). Conversely, if Eq. (39) holds, ξ_i^B and ξ_j^B share common eigenvectors for any i and j. By choosing these eigenvectors as the basis for von Neumann measurement, the state does not change after the measurement, which means that ξ_{AB} is a quantum-classical state. Now consider an arbitrary quantum-classical state in form of Eqs. (38) and (39) as the input state, the channel Λ acting on subsystem B leads the state to $\xi'_{AB} \equiv \mathrm{I}_A \otimes \Lambda_B(\xi_{AB}) = \sum_i p_i \xi_i^A \otimes \Lambda(\xi_i^B)$, which is still a quantum-classical state if and only if

$$[\Lambda(\xi_i^B), \Lambda(\xi_j^B)] = 0, \tag{41}$$

for arbitrary choice of ξ_i^B and ξ_j^B satisfying Eq. (39). This is just the definition of a commutativity-preserving channel. Therefore, the channel Λ can create quantum correlation for some input quantum-classical states if and only if it is not a commutativity-preserving channel. This completes the proof of the theorem.

It is worth mentioning that a channel Λ is commutativity-preserving if and only if

$$[\Lambda(|\phi\rangle), \Lambda(|\psi\rangle)] = 0 \tag{42}$$

holds for any pure states satisfying $\langle\phi|\psi\rangle = 0$. This criterion is equivalent to Eq. (37). The "only if" part is obtained directly by choosing $\xi = |\phi\rangle\langle\phi|$ and $\xi' = |\psi\rangle\langle\psi|$ in Eq. (37). Conversely, if Eq. (42) holds, by writing ξ and ξ' on their common eigenbasis, we arrive at Eq. (37).

For qubit case, a commutativity-preserving channel is either a unital channel or a completely decohering channel. In order to prove this statement, we notice that a qubit channel is commutativity-preserving if and only if

$$[\Lambda(\mathrm{I}), \Lambda(\rho)] = 0, \forall\rho. \tag{43}$$

The "only if" part is obvious from definition since $[\mathrm{I}, \rho] = 0, \forall\rho$. For the "if" part, by using the linearity of Λ, the left hand side of Eq. (42) can be written as

$$\frac{1}{2}[\Lambda(|\phi\rangle\langle\phi| + |\psi\rangle\langle\psi|), \Lambda(|\psi\rangle\langle\psi| - |\phi\rangle\langle\phi|)] = \frac{1}{2}[\Lambda(\mathrm{I}), \Lambda(u\sigma^z u^\dagger)], \tag{44}$$

where $|\psi\rangle = u|0\rangle$, $|\phi\rangle = u|1\rangle$, and u is a qubit unit operator. Since any qubit state ρ can be decomposed as $\rho = (\mathrm{I} + n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)/2$, Eq. (42) is equivalent to Eq. (43). From this observation, a qubit channel Λ is commutativity-preserving if and only if it is one of the following two cases: Case 1: $\Lambda(\mathrm{I}) = \mathrm{I}$, which means that Λ is a unital channel. Case 2: $\Lambda(\mathrm{I}) \neq \mathrm{I}$, and consequently, the channel Λ must take any input state ρ to a diagonal form on the eigenbasis of $\Lambda(\mathrm{I})$, and is thus a completely decohering channel. Therefore, when B is a qubit, a local channel that can create quantum correlations on B if and only if it is neither a unital channel nor a completely decohering channel.

For qutrit case, even unital channel can create quantum correlations. The necessary and sufficient condition for a commutativity-preserving qutrit channel is that it is either an isotropic channel or a completely decohering channel. This result has been generalized to channels whose dimension $d \geqslant 3$ is finite.

5.2.3. Quantum correlating power of local quantum channels

As we have discussed, a local channel Λ acting on subsystem B can drive a quantum-classical state into a state with positive quantum correlation on B. In order to quantify the amount of quantum correlation that is induced by Λ, we give the definition of quantum-correlating power[22].

Definition (quantum-correlating power). The quantum-correlating power of a quantum channel is defined as

$$\mathcal{Q}(\Lambda) = \max_{\rho\in\mathcal{C}_0} Q(I \otimes \Lambda(\rho)), \tag{45}$$

where Q is a measure of quantum correlation santisfying the three conditions in Sec. 2.2. Quantum discord and one-way quantum deficit are examples of such quantum correlation measure. The quantum-correlating power has the following properties.

(a) Optimal input states. For any d-dimension local channel acting on B, the optimal input classical state with the maximum amount of quantum correlation in the output state is a classical-classical state of form

$$\varrho = \sum_{j=0}^{d-1} q_j \Pi_{\alpha_j}^A \otimes \Pi_{\beta_j}^B, \tag{46}$$

where $\Pi_{\alpha_j}^A = |\alpha_j\rangle\langle\alpha_j|$ are orthogonal basis for the Hilbert space of qudit A. This property greatly simplifies the maximization for calculation of QCP.

(b) The local single-qubit channel with maximum QCP can be found in the set of channels

$$\mathcal{D}_0 = \{\Lambda|\Lambda(\cdot) = \sum_{i=0}^{1} E_i(\cdot)E_i^\dagger, E_i = |\psi_i\rangle\langle\alpha_i|\}, \tag{47}$$

where $|\psi_0\rangle$ and $|\psi_1\rangle$ are two non-orthogonal pure states.

Based on this property, the local single-qubit channel with the maximum QCP based on quantum discord is $\tilde{\Lambda}(\cdot) = \sum_{i=0}^{1} \tilde{E}_i(\cdot)\tilde{E}_i^\dagger$, where $\tilde{E}_0 = |0\rangle\langle 0|, \tilde{E}_1 = |+\rangle\langle 1|$, and the corresponding QCP is $\mathcal{Q}_\delta(\Lambda^{Max}) = 2h(\frac{1}{\sqrt{2}}) - 1 \approx 0.2017$. It is worth mentioning that there are separable states containing larger quantum discord. An example is $\rho = (|\Phi^+\rangle\langle\Phi^+| + |\Phi^-\rangle\langle\Phi^-|)/4 + |\Psi^+\rangle\langle\Psi^+|/2$, whose quantum discord is $\delta(\rho) = 3(2 - 3\log_2 3)/4 \approx 0.311$. Such states can not be prepared by local operations from a single copy of two-qubit classical state.

(c) When two channels Λ_1 and Λ_2 are used paralleled, the discord-based QCP of the composite channel $\Lambda_1 \otimes \Lambda_2$ is no less than the sum of the QCP for the two channels

$$\mathcal{Q}(\Lambda_1 \otimes \Lambda_2) \geqslant \mathcal{Q}(\Lambda_1) + \mathcal{Q}(\Lambda_2), \tag{48}$$

which is called the super-additivity of QCP[23].

This phenomena holds for any finite dimension channels. It is worth mentioning that the channels satisfying $\mathcal{Q}(\Lambda_1 \otimes \Lambda_2) > \mathcal{Q}(\Lambda_1) + \mathcal{Q}(\Lambda_2)$ are quite common. Here we give an example of phase-damping qubit channels, whose Kraus operators are $E_0^{\rm PD} = |0\rangle\langle 0| + \sqrt{1-p}|1\rangle\langle 1|$ and $E_1^{\rm PD} = \sqrt{p}|1\rangle\langle 1|$. Clearly, $\mathcal{Q}(\mathrm{PD}) = 0$. Now consider initial state of qubits A and B $\rho_{AB} = \frac{1}{2}\sum_{i=0}^{1} |i\rangle_A\langle i| \otimes |i\rangle_B\langle i|$. Qubits A' and B' are in the same state, then the total state of the four qubits is

$$\rho = \rho_{AB} \otimes \rho_{A'B'} = \frac{1}{4}\sum_{i,j} |ij\rangle_{AA'}\langle ij| \otimes |ij\rangle_{BB'}\langle ij|. \tag{49}$$

Now apply a two-qubit unitary operation U: $U|ij\rangle = |\psi_{ij}\rangle$ on qubits A and A', where $|\psi_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, $|\psi_{11}\rangle = \frac{1}{\sqrt{2}}(|0+\rangle + |1-\rangle)$, $|\psi_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, and $|\psi_{10}\rangle = \frac{1}{\sqrt{2}}(|0-\rangle - |1+\rangle)$. Then qubits A and A' each transmits through a PD channel, and the output state becomes $\rho' = \Lambda_A^{\rm PD} \otimes \Lambda_{A'}^{\rm PD} \otimes I_{BB'}(U_{AA'}\rho U_{AA'}^\dagger)$. Now we check whether quantum correlation defined on AA' is created between the bipartition $AA' : BB'$ by using Eq. (42). Notice that $[\Lambda^{\rm PD} \otimes \Lambda^{\rm PD}(\psi_{00}), \Lambda^{\rm PD} \otimes \Lambda^{\rm PD}(\psi_{11})] = \frac{1}{8}ip\sqrt{1-p}(\sigma^y \otimes \sigma^z + \sigma_z \otimes \sigma^y) \neq 0$, and consequently, quantum correlation is created between the bipartition $AA' : BB'$.

6 Conclusion

Quantum correlation accounts for the nonlocal coherence between quantum systems. The fact that it can be increased or even created by quantum decoherence process is counter intuitive and thus attract much research interest. This phenomenon depends on both initial state and the type of local quantum channels. For pure initial states, local quantum noise can never increase the quantum correlation. The optimal input zero-discord state, which associate with the maximum output quantum correlation for any local channels, is the classical-classical state. The class of initial bipartite states whose quantum correlation can not be increased is still an open and interesting question.

The origin for noise increased quantum correlation is that local quantum channels can turn classical correlations into quantum ones. The possibility of this process comes from the definition of classical-quantum state, which relies on the choice of basis. Measurement on other basis may change the state of the bipartite system, so the nonlocal coherence is actually hidden in the classical states. Local quantum channels reveal the coherence and thus generate the quantum correlation. From this point of view, the ability of quantum channel to create quantum correlation represents the 'quantumness' of the channel. The quantum correlating power is inherent to local quantum channels and possesses properties such as superadditivity.

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